# $f$-state pairing in superfluid $^3$He-$A_1$

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A dipole interaction in $^3$He-$A_1$ leads to an induced superfluidity of particles whose spin is oriented along the magnetic field. In a parallel-plane geometry these particles become paired into either a $p$ state or an $f$ state, depending on the texture of the orbital vector **l**. Reversing the magnetic field causes a transition from one state to another.

The transition of liquid $^3$He into a superfluid state in an external magnetic field is known to involve an initial transition to the $A_1$ phase, where the only fermions that are paired are those whose spin is antiparallel to the field **H**, so that the magnetic moment is directed along **H** (Refs. 1 and 2). A weak spin-orbit (dipole) interaction mixes the states with the different spin projections, with the result that an energy gap $\Delta_\uparrow$ is also induced for the spin-up quasiparticles (those whose spin is oriented along the field).[3] Although the induced gap $\Delta_\uparrow$ is several orders of magnitude smaller than the gap of the majority carriers of the superfluid current, $\Delta_\downarrow$, MHD experiments carried out at a low frequency $\omega < \Delta_\uparrow$ can detect the existence of spin-up Cooper pairs.[4]

In the present letter we analyze the induced superfluidity of spin-up fermions in the case of a parallel-plane geometry, with plates separated by a distance smaller than the dipole length $\xi_d \sim 10^{-3}$ cm. We will show that, depending on the orientation of the orbital vector **l** with respect to **H**, the pairing of the minority carriers in $^3$He-$A_1$ (spin-up fermions) occurs into either a $p$ state or an $f$ state.

If we ignore the dipole interaction, we would describe Cooper pairs in $^3$He-$A_1$ by means of two quantum numbers: the spin projection, $S_z = -1$, onto the magnetic field ($\mathbf{H}\|\hat{\mathbf{z}}$) and the projection of the orbital angular momentum of the relative motion of the atoms in the Cooper pair. Let us assume that the plates bracketing the $^3$He-$A_1$ are oriented perpendicular to the magnetic field. The unit vector **l**, which determines the direction of the orbital angular momentum, is then oriented, by virtue of the boundary conditions, either along or opposite **H**. The projection of the orbital angular momentum onto the magnetic field, $L_z$, is therefore 1 in the case $\mathbf{l} = \hat{\mathbf{z}}$ and $-1$ in the case $\mathbf{l} = -\hat{\mathbf{z}}$.

The dipole interaction has the consequence that neither $S_z$ nor $L_z$ is conserved separately; only the projection of the total angular momentum, $J_z = L_z + S_z$, is conserved. A state with $J_z = 0$ occurs in the case $\mathbf{l} = \hat{\mathbf{z}}$, while a state with $J_z = -2$ occurs in the case $\mathbf{l} = -\hat{\mathbf{z}}$. Consequently, the dipole interaction gives rise to an admixture of Cooper pairs with $S_z = 0$ and $S_z = 1$; in a state with $J_z = 0$, they correspond to orbital-angular-momentum projections $L_z = 0$ and $L_z = -1$. In a state with $J_z = -2$, they correspond to the projections $L_z = -2$ and $L_z = -3$, which are possible only if the orbital angular momentum takes on a value $L = 3$ or higher.

In the case $\mathbf{l} = \hat{\mathbf{z}}$ the pairing of minority carriers (with $S_z \neq -1$) thus occurs into a $p$ state, while in the case $\mathbf{l} = -\mathbf{z}$ and $f$ state necessarily arises. This circumstance sets the present case apart from that of a free geometry,[3] where **l** is oriented perpendicular to the magnetic field, so the angular momenta do not combine in a simple way. As a result, the pairing of the minority carriers always occurs into a $p$ state. Although an $f$ state in superfluid $^3$He-$A_1$ has been discussed previously, it has been treated as an admixture to a $p$ state which arises far from $T_c$ because of nonconservation of the quantum number $L$ in the nonlinear Gor'kov equations.[5,6] In the case at hand, an $f$ state arises as a pure state near $T_c$ on the Fermi surface of the minority carriers; far from $T_c$, this state contains admixtures of states with higher angular momenta: $L = 5, 7, \ldots$ .

What is the size of the gap $\Delta_\uparrow$ induced at the Fermi surface of the spin-up quasiparticles in a state with $J_z = -2$? Since $\Delta_\uparrow$ is dominated by the $S_z = 1$ state, we consider in the order parameter—a symmetric spinor $\Delta_{\alpha\beta}(\mathbf{k})$, which depends on the momentum of the paired quasiparticles—only a superposition of two states, ($S_z = -1$, $L_z = -1$), ($S_z = 1$, $L_z = -3$):

$$\Delta_{\alpha\beta}(\mathbf{k}) = i(\sigma_y\vec{\sigma})_{\alpha\beta}(\mathbf{d}_\downarrow(\mathbf{n}) + \mathbf{d}_\uparrow(\mathbf{n})), \quad \mathbf{n} = \mathbf{k}/k_F \ , \tag{1}$$

where $\vec{\sigma}$ are the Pauli matrices; the vectors **d** are given by

$$\mathbf{d}_\downarrow(\mathbf{n}) = \Delta_\downarrow \mathbf{e}^*(\mathbf{e}^*\cdot\mathbf{n}), \quad \mathbf{d}_\uparrow(\mathbf{n}) = \Delta_\uparrow^f \mathbf{e}(\mathbf{e}^*\cdot\mathbf{n})^3, \quad \mathbf{e} = \hat{\mathbf{x}} + i\hat{\mathbf{y}} \ , \tag{2}$$

$|\Delta_\downarrow|$ and $|\Delta_\uparrow^f|$ are the corresponding amplitudes of the energy gap at each of the Fermi surfaces, and the superscript $f$ means that the gap arises because of pairing in an $f$ state.

The Ginzburg-Landau functional for $\Delta_\downarrow$ and $\Delta_\uparrow^f$ is

$$F_{GL} = -\alpha|\Delta_\downarrow|^2 + \beta|\Delta_\downarrow|^4 + \lambda_d(\Delta_\downarrow\Delta_\uparrow^{f*} + \Delta_\downarrow^*\Delta_\uparrow^f) + \alpha_f|\Delta_\uparrow^f|^2 . \qquad (3)$$

The first two terms, with $\alpha \sim N_F(1 - T/T_{c1})$ and $\beta \sim N_F/T_c^2$ ($N_F$ is the state density, and $T_{c1}$ is the temperature of the transition to the $A_1$ phase), correspond to a functional for pure $p$ pairing; the third term, with $\lambda_d \sim (\xi_0/\xi_d)^2 N_F$ ($\xi_0$ is the coherent length at $T = 0$), describes a dipole interaction of oppositely directed spins. This term is found by averaging the magnetic-dipole interaction of the $^3$He atoms over state (1), as was done in the review by Leggett.[1] The last term, with $\alpha_f \sim N_F$, describes the positive energy of the $f$ state.[6]

A minimization of expression (3) with respect to $\Delta_\downarrow$ and $\Delta_\uparrow^f$ leads to the following estimate of the amplitude of the gap in the $f$ state:

$$\Delta_\uparrow^f \sim \left(\frac{\xi_0}{\xi_d}\right)^2 \Delta_\downarrow \sim 10^{-5}\Delta_\downarrow , \qquad \Delta_\downarrow \sim T_c\left(1 - \frac{T}{T_{c1}}\right)^{1/2} . \qquad (4)$$

This gap is smaller by an amount $1 - T/T_{c1}$ than the gap $\Delta_\uparrow^p$ in the $p$ state, which arises in the case $J_z = 0$, i.e., with $\mathbf{l}\|\hat{\mathbf{z}}$:

$$\Delta_\uparrow^p \sim \left(\frac{\xi}{\xi_d}\right)^2 \Delta_\downarrow \qquad \xi \sim \xi_0\left(1 - \frac{T}{T_{c1}}\right)^{-1/2} . \qquad (5)$$

A transition from the $p$ state to the $f$ state and back can be arranged in the given texture of the vector $\mathbf{l}$ by rotating the magnetic field through 180°. The states can be identified by, for example, MHD experiments capable of distinguishing between states with $J_z = 0$ and $J_z = -2$. In a state with $J_z = -2$, the relative gauge-rotation symmetry is broken. Specifically, order parameter (1),(2) changes both under a gauge transformation and when the vector $\mathbf{e}$ is rotated around the $z$ axis. It is not possible to distinguish between the effects of these transformation, however, since each leads to a multiplication of the order parameter by $e^{i\alpha}$. According to Ref. 7, this symmetry breaking should lead to a magnetothermomechanical effect. In a state with $J_z = 0$, only the gauge symmetry is broken, so that such an effect should not occur at low frequencies $\omega < \Delta_\uparrow$.

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