Superfluid transport of precession in ^{3}He-B

É. B. Sonin
A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

(Submitted 30 April 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 12, 586–589 (25 June 1987)

The superfluid transport of precession which was recently observed in the B phase is analyzed. The Landau critical current is found. The structure of the precession vortex which arises during phase slippage of the precession is discussed.

Experimental[1] and theoretical[2] studies which were recently reported indicate the existence of a magnetic superfluidity in the B phase of He^3. "Magnetic superfluidity" means the transport of spin over macroscopically large distances by a dissipationless flow (superflow) which is proportional to the angle through which the order parameter rotates in spin space. With regard to relaxation processes in the A phase of He^3, this phenomenon has been under discussion since the papers by Vuorio[3] and Corruccini *et al.*[4] The analogy between dissipationless spin transport and superfluid mass transport is a limited one because spin, in contrast with mass, does not obey a strict conservation law. The difference has a substantial effect on the nature of the dissipationless transport, but it does not rule out the possibility that it exists and can be observed.[5]

The dissipationless transport observed in the B phase[1,2] has several features which distinguish it from the dissipationless spin transport discussed previously. While the previous discussion[5] dealt with the transport of the projection of the spin onto the axis of a static magnetic field, in the B phase we are dealing with the transport of a physical quantity which is equal to the difference between the projection of the spin onto the axis of the magnetic field and its projection onto some moving axis which nearly coincides with the spin vector that is precessing around the magnetic field.[1)] This quantity is an approximate integral of motion (an exact integral in the limit of vanishing gradients[6]). We will refer to it as the "precession moment," since the quantity which is its canonical conjugate is not a rotation angle of the order parameter in spin space, as it would be for a genuine spin, but instead the rotation angle of a precessing spin (a precession phase). We can therefore speak in terms of two types of superfluid transport in spin dynamics: transport of spin and transport of precession moment (or transport of precession).

In this letter we calculate the critical gradient of the precession phase, above which the precession superflow looses its stability in the Landau sense: The creation of spin waves reduces the energy of the system. The critical gradient is equal in order of magnitude to the reciprocal of the dipole length $\sim 10^3$ cm^{-1} if the angle (β) between the precessing spin and the magnetic field exceeds 104°. At $\beta < 104°$, it vanishes.

The free energy of spin precession for the case of gradients perpendicular to the magnetic field is, according to Fomin,[2,6]

$$F = \frac{\chi}{\gamma^2}\left\{(\omega_P - \omega_L)\omega_L(u-1) + \frac{1}{2}[(1-u)^2 c_\parallel^2 + (1-u^2)c_\perp^2](\vec{\nabla}\alpha)^2 + \frac{1}{2}c_\parallel^2(\vec{\nabla}\phi)^2 - c_\parallel^2(1-u)\vec{\nabla}\alpha\cdot\vec{\nabla}\Phi + \frac{1}{2}c_\perp^2\frac{(\vec{\nabla}u)^2}{1-u^2}\right\} + V(u,\Phi), \tag{1}$$

where χ is the susceptibility; γ is the gyromagnetic ratio; $c_\parallel$ and $c_\perp$ are the velocities of spin waves; α, Φ, and β are the Euler angles in Fomin's theory (α is the precession phase); and $\omega_L = \gamma H$ is the Larmor frequency. The precession frequency ω_p is a Lagrange multiplier, which minimizes the free energy for a given precession moment with a density $P = M_z - M = (u-1)\omega_L\chi/\gamma$. The angle Φ is determined in a homogeneous state from the condition for a minimum of the dipole energy,

$$V(u,\Phi) = \frac{2}{15}\frac{\chi\Omega^2}{\gamma^2}\left[(1+\cos\Phi)u + \cos\Phi - \frac{1}{2}\right]^2. \tag{2}$$

It vanishes at $\beta > 104°$. At $\beta < 104°$, a minimization of V with respect to Φ results in the vanishing of V. From Hamilton's equations for the pair of conjugate variables α, P we find $\vec{\nabla}\Phi = \vec{\nabla}u = 0$ for a steady state with a precession flux

$$\mathbf{j}_P = -\gamma\frac{\partial F}{\partial\vec{\nabla}\alpha} = -\gamma A(u)\vec{\nabla}\alpha = -\frac{\chi}{\gamma}[(1-u)^2c_\parallel^2 + (1-u^2)c_\perp^2]\vec{\nabla}\alpha. \tag{3}$$

The value of u is found from the condition for a minimum of free energy (1):

$$\frac{\partial F}{\partial u} = \frac{\chi}{\gamma^2}(\omega_P - \omega_L)\omega_L + \frac{1}{2}A'(u)(\vec{\nabla}\alpha)^2 + V'(u) = 0. \tag{4}$$

This condition is equivalent to the condition that the chemical potential for the steady-state superfluid mass flux remains constant.

To determine the stability of the precession flux, we find the change in the energy of a state with a precession flux which is caused by a small static plane-wave ($\sim e^{ikx}$) fluctuation of its parameters. For small values of k, all the corrections which arise from the terms containing $\nabla\Phi$ and ∇u can be discarded, and the fluctuation energy,

$$\delta F = A(u_0)\frac{(\vec{\nabla}\alpha')^2}{2} + A'(u_0)\mathbf{h}\cdot\vec{\nabla}\alpha' u' + [A''(u_0)\frac{\mathbf{h}^2}{2} + V''(u_0)]\frac{u'^2}{2}, \qquad (5)$$

is quadratic in the small deviations $u' = u - u_0$ and $\vec{\nabla}\alpha' = \vec{\nabla}\alpha - \mathbf{h}$ from the steady current state, with given $u = u_0$ and $\vec{\nabla}\alpha = \mathbf{h}$ (we will be omitting the subscript 0). The condition that this quadratic form be positive definite,

$$A(u)[A''(u)h^2 + 2V''(u)] > 2A'(u)^2 h^2 , \qquad (6)$$

is the stability condition. The value of h at which this condition is first violated is the critical value of the gradient; at $\beta > 104°$, it is given by

$$h_{\rm cr} = \left\{\frac{A(u)\,V''(u)}{A'(u)^2 - A(u)A''(u)/2}\right\}^{1/2} = \frac{4\Omega}{3\sqrt{5}}\left\{\frac{c_\parallel^2(1-u)^2 + c_\perp^2(1-u^2)}{[c_\parallel^2 + (c_\perp^2 - c_\parallel^2)u]^2 + c_\perp^4/3}\right\}^{1/2}. \qquad (7)$$

At $\beta < 104°$ we have $V''(u) = 0$, from which we find $h_{\rm cr} = 0$. Consequently, at $\beta = 104°$ there is an abrupt change in the critical gradient from 0 to a value on the order of the reciprocal of the dipole length, $\xi_D^{-1} \sim \Omega/c$.

In Fomin's study,[2] the critical value was determined by the length $\xi \approx c/\sqrt{(\omega_P - \omega_L)\omega_L}$, which does not—according to the analysis above—directly affect the stability, since the term $\sim(\omega_P - \omega_L)$ in free energy (1) is linear in $P \sim (u-1)$ and does not contribute to the quadratic form which determines the stability. However, $\omega_P - \omega_L$ does affect the value of u in the flow, and a change in u may

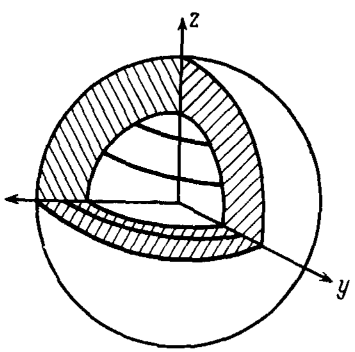

FIG. 1. Mapping of states with a uniform precession flux onto the space of the order parameter of the B phase (a sphere of radius π). Part of the sphere is cut away to show the sphere of radius $\cos^{-1}(1/4)$ (an angle of 104°), on which the dipole energy vanishes (degenerate ground states). The thick solid lines are mappings of states with a precession flux onto the sphere of angle 104° ($\beta < 104°$) and onto the xy plane ($\beta > 104°$).

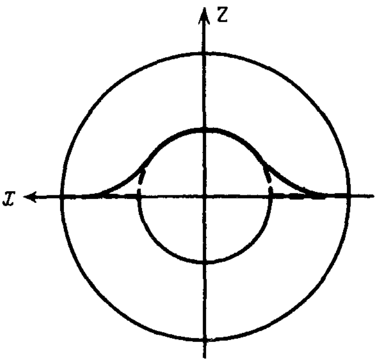

FIG. 2. Mapping of a precession vortex in a section in the xz plane. Dashed line—section of a surface of the mapping for homogeneous fluxes; thick solid line—the same, for a vortex. The complete mapping surfaces are surfaces of revolution for the indicated lines around the z axis.

cause a transition from a large critical gradient to a vanishing one (see the discussion of the experiments at the end of this letter).

The condition for the stability of a superfluid precession flow which was derived above is an analog of the Landau criterion (cf. the derivation of the analogous criterion for a spin flux in a ferromagnetic, which is summarized at the end of Subsection 3d in Ref. 5). From the theory of superfluidity we know that dissipation in a superflow sets in before the Landau critical velocities are reached (experimentally, the dissipation sometimes sets in considerably earlier) and is determined by a process of vortex formation and the motion of vortices across the flow (phase slippage). The structure of an axisymmetric vortex which causes a slippage of the precession phase (a precession vortex) is determined by the circumstance that the gradient of the phase, $\nabla\alpha$, has only an azimuthal component $\nabla\alpha = 1/r$ (r is the distance from the vortex line), while Φ and $u = \cos\beta$ depend on only r and are found by minimizing the energy. At the center of a vortex we have $u = 1$ and $\Phi = 104°$, while at the periphery the angle Φ exponentially approaches zero on the line ξ_D. The expression describing the decay of the deviation, $u' = u - u_\infty$ (u_∞ is the value of u as $\rho \to \infty$), on the other hand, is a power law:

$$u' \doteq \frac{15}{16}\,\frac{c_\parallel^2(1-u) + c_\perp^2 u}{\Omega^2}\,\frac{1}{r^2}\,.$$

Figure 1 shows a mapping of states with a uniform precession flux onto the space of the order parameter of the B phase [the space of three-dimensional rotations $R(\mathbf{n},\theta)$], which is a sphere of radius π. The points of this sphere are found by plotting the rotation angle θ along the direction of the directrix $\mathbf{n}$ (the rotation axis) drawn from the center of the sphere. It can be seen from Fig. 1 that in a state with a precession flux there is a spatial rotation of $\mathbf{n}$ around the z axis. Figure 2 shows a section through the mapping of a precession vortex.

In the experiments of Ref. 1, phase slippage was observed at phase gradients an order of magnitude below the critical value found above, $\xi_D^{-1} \simeq 10^3\ \mathrm{cm}^{-1}$. In addition to the general factors which stem from the aspects of vortex formation that are still unclear, even for an ordinary superfluid liquid, this discrepancy might be explained in the following way. In the experiment, the angle β was just slightly above 104°. It follows from condition (4), which determines $u = \cos\beta$, that an increase in $\nabla\alpha$ at a given precession rate ω_p leads to an increase in u, i.e., to a decrease in β. Locally, in regions of a large gradient $\nabla\alpha$, the angle β may fall below 104°, and in such regions a phase slippage will begin immediately since the critical gradient vanishes at $\beta < 104°$.

I am indebted to I. A. Fomin for useful discussions.

[1)]V. L. Golo brought this to our attention.

[1]A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev, and Yu. M. Mukharskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 98 (1987) [JETP Lett. **45**, 124 (1987)].

[2]I. A. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 106 (1987) [JETP Lett. **45**, 135 (1987)].

[3]M. J. Vuorio, J. Phys. **C7**, L5 (1974).

[4]L. R. Corruccini, D. D. Osheroff, D. M. Lee, and R. C. Richardson, Phys. Rev. Lett. **34**, 564 (1975).

[5]É. B. Sonin, Usp. Fiz. Nauk **137**, 267 (1982) [Sov. Phys. Usp. **25**, 409 (1982)].

[6]I. A. Fomin, Zh. Eksp. Teor. Fiz. **88**, 2039 (1985) [Sov. Phys. JETP **61**, 1207 (1985)].

Translated by Dave Parsons