Nonlinear susceptibility of an Ising spin glass in a transverse field

S. I. Berim and R. V. Saburova
Kazan Branch of the Moscow Energetics Institute, 420066, Kazan

(Submitted 24 June 1991; resubmitted 25 July 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **54**, No. 5, 287–290 (10 September 1991)

Nonlinear susceptibilities are calculated for the Sherrington–Kirkpatrick model of a spin glass in a transverse field with a shifted Gaussian distribution of exchange couplings. The phase transition temperatures and the critical values of the transverse field are found.

Spin glasses in a transverse field have recently attracted particular research interest. The free energy, the temperature at which the spins "freeze" into the spin-glass phase (T_{sg}), and the critical value of the transverse field (Γ_{cr}), above which the phase transition to the spin-glass state is cut off, have been calculated for the Ising model of a spin glass in a transverse field with an infinite range (the Sherrington–Kirkpatrick model) (Refs. 1–6, for example). Theoretical work has been stimulated on proton glasses (a mixture of ferroelectric and antiferroelectric materials), certain solid solutions, and tunnel dipole glasses (alkali halide crystals and virtual ferroelectrics activated by tunnel dipole centers).[7–10] In such systems, the role of the transverse field is played by a tunneling of pseudospins. In such systems one observes "spin-glass" properties, and whether there is a clearly defined phase transition or a gradual freezing of pseudospins is still an open question. We know that the divergence of the nonlinear susceptibility in a spin glass characterizes a phase transition.[11–13] In this letter we carry out the first calculations of the nonlinear susceptibilities of an Ising model of a spin glass in a transverse field Γ for various phase transitions. We analyze the results. We consider an exchange interaction of infinite range, J_{ij}, with a normal distribution with a nonzero mean value J_0/N and a variance J^2/N (N is the number of spins in the system).[14] The Hamiltonian of the problem is

$$\mathcal{H} = -\sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z - h\sum_{i=1}^{N}\sigma_i^z - \Gamma\sum_{i=1}^{N}\sigma_i^x, \tag{1}$$

where σ_i^α ($\alpha = x, y, z$) are the Pauli matrices of spin i, and h is the applied magnetic field. The order parameters m (the magnetization per spin) and q (the order parameter of the spin-glass phase) for this model are[3,6]

$$m = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dz e^{-z^2/2} R W^{-1} \tanh \beta W,$$

$$q = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dz e^{-z^2/2} R^2 W^{-2} \tanh^2 \beta W, \tag{2}$$

where $R = J_0 m + Jz\sqrt{q} + h$, $W^2 = R^2 + \Gamma^2$, and $\beta = (k_B T)^{-1}$. According to Ref. 6, three phases are clearly identifiable, depending on the relations among J_0, J, and Γ: a paramagnetic phase (P), with $m = 0$ and $q = 0$; a ferromagnetic phase (F), with $m \neq 0$ and $q \neq 0$; and a spin-glass phase (SG), with $m = 0$ and $q \neq 0$. To calculate the susceptibilities χ_n ($\chi_n = \lim_{h \to 0} \partial^{n+1} m / \partial h^{n+1}$) in the case of a low field h, we expand m and q in Taylor series:[15]

$$m = m_0 + \chi_0 h + \chi_1 h^2 + \chi_2 h^3 + \ldots, \quad q = q_0 + q_1 h + q_2 h^2 + q_3 h^3 + \ldots,$$
$$(n = 0, 1, 2, \ldots). \tag{3}$$

Using (3) in Eqs. (2), we can find m_0, q_0, χ_n, and q_n in succession, and we can express the susceptibilities in terms of m_0 and q_0. These quantities are in turn determined from self-consistent equations of the type

$$m_0 = < R_0 W_0^{-1} \tanh \beta W_0 >, \quad q_0 < R_0^2 W_0^{-2} \tanh^2 \beta W_0 >, \tag{4}$$

where

$$A(z) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dz e^{-z^2/2} A(z), \quad R_0 = J_0 m_0 + Jz\sqrt{q_0}, \quad W_0^2 = R_0^2 + \Gamma^2.$$

We have calculated general expressions for the susceptibilities χ_0, χ_1, and χ_2, but they are very lengthy, and we will not reproduce them here. Let us analyze the temperature dependence near each of the phase boundaries. For the P↔SG phase transition, the linear susceptibility is

$$\chi_0(\mathrm{P} \to \mathrm{SG}) = \Gamma^{-1} \tanh \beta\Gamma \{1 - J_0 \Gamma^{-1} \tanh \beta\Gamma\}^{-1}, \quad \tilde{W}_0^2 = J^2 z^2 q_0 + \Gamma^2, \tag{5}$$

$$\chi_0(\mathrm{SG} \to \mathrm{P}) = \frac{\beta\{(1 - q_0) - \Gamma^2 < W_0^{-2} >\} + \Gamma^2 < \tilde{W}_0^{-3} \tanh \beta \tilde{W}_0 >}{1 - J_0[\beta\{(1 - q_0) - \Gamma^2 < \tilde{W}_0^{-2} >\} + \Gamma^2 < \tilde{W}_0^{-3} \tanh \beta \tilde{W}_0 >]}. \tag{6}$$

The nonlinear susceptibility χ_1 for the P↔SG transition is zero. The nonlinear susceptibility χ_2 is

$$\chi_2(\mathrm{P} \to \mathrm{SG}) = -\chi_0^4(\mathrm{P} \to \mathrm{SG}) \frac{A}{B} [\beta\Gamma(1 - \tanh^2 \beta\Gamma)$$
$$- \tanh \beta\Gamma] 2^{-1} \Gamma^{-3}) \Gamma^{-1} \tanh \beta\Gamma)^{-4},$$
$$\chi_2(\mathrm{SG} \to \mathrm{P}) \approx \chi_0^4(\mathrm{SG} \to \mathrm{P}) \beta^{-1} A B^{-1}; A = 1 + 2J^2 \Gamma^{-2} \tanh^2 \beta\Gamma,$$
$$B = 1 - \Gamma^{-2} J^2 \tanh^2 \beta\Gamma. \tag{7}$$

We find an equation for $T_{sg}(\Gamma)$ by setting $B = 0$ in (7). The result is

$$\tanh \frac{\Gamma}{k_B T_{sg}(\Gamma)} = \frac{\Gamma}{J}. \tag{8}$$

The same equation for $T_{sg}(\Gamma)$ was derived in Refs. 3, 5, and 6. It follows from (5)–(8) that at the temperature $T_{sg}(\Gamma)$ the susceptibility χ_0(P↔SG) has a change in slope, while the susceptibility $\chi_2(\text{P}\leftrightarrow\text{SG})$ diverges negatively on each side of the phase transition, with $\chi_2 \sim |T - T_{sg}(\Gamma)|^{-1}$. We have calculated the static macroscopic susceptibilities in (5)–(7) for the first time in the case $J_0 \neq 0$. The susceptibilities χ_0 and χ_2 found in Ref. 2 for the case $J_0 = 0$ (for the P↔SG phase transition) and by a perturbation theory in the transverse field Γ exhibit a similar behavior near the temperature of this phase transition, $T_{sg}(\Gamma)$. The slope change in χ_0 was found for $J_0 = 0$ in Refs. 3 and 6. It follows from (8) that the transverse field reduces the phase-transition temperature $T_{sg}(\Gamma)$ to a point below $T_{sg}(k_B T_{sg} = J)$ in the absence of a transverse field. The decrease depends on the relation between Γ and J. The critical value of the transverse field, Γ_{cr}^{sg}, above which a phase transition does not occur, is, according to (8),

$$\Gamma_{cr}^{sg}(T_{sg}(\Gamma) = 0) = J. \tag{9}$$

This value is the same as the value of Γ_{cr}^{sg} in Refs. 3, 5, and 6, while it differs by a factor of about 1.5 from Γ_{cr}^{sg} in Ref. 2.

Analysis of the susceptibilities for the P↔F phase transition shows that χ_0 and χ_2 diverge. The temperature of the phase transition, $T_c(\Gamma)$, and Γ_{cr}^{c} are given by the expressions

$$\tanh \frac{\Gamma}{k_B T_c(\Gamma)} = \frac{\Gamma}{J_0}, \quad \Gamma_{cr}^{c}(T_c(\Gamma) = 0) = J_0. \tag{10}$$

In the case of the SG↔F phase transition, both the linear and nonlinear susceptibili-

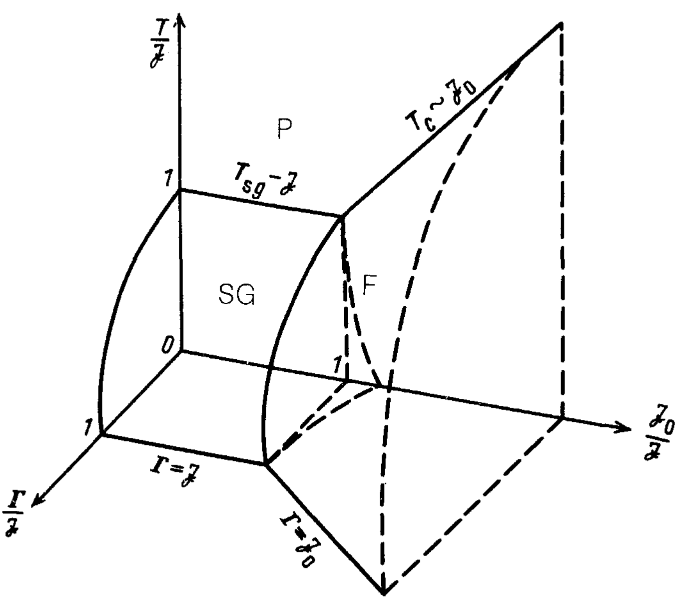

FIG. 1. Schematic phase diagram of an infinite-range Ising model of a spin glass in a transverse field. The notation is explained in the text proper.

ties diverge. The temperature of this phase transition, $T_f(\Gamma)$, is

$$k_B T_f(\Gamma) \approx J_0(1 - q_0). \tag{11}$$

Figure 1 shows a (schematic) phase diagram of our model of a spin glass in a transverse field (in units of J), as described by expression (1) and a Gaussian distribution of exchange couplings with a nonzero mean value. It can be seen from (8), (10), and (11) that the critical temperatures decrease as a result of the transverse field. In the absence of a transverse field, the phase-transition temperatures and the susceptibilities are the same as in Ref. 15. The phase transition smooths out the change in slope on the linear susceptibility, and the $\chi_0(T)$ curve shifts down the temperature scale. The transverse field prevents the divergence of χ_2. At values of the transverse field above Γ_{cr}, phase transitions become impossible.

[1] Ya. V. Fedorov and E. F. Shender, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 526 (1986) [JETP Lett. **43**, 681 (1986)].
[2] T. Yamamoto and H. Ishii, J. Phys. C **20**, 6053 (1987).
[3] T. K. Kopec, J. Phys. C **21**, 6053 (1988).
[4] S. L. Ginzburg, Zh. Eksp. Teor. Fiz. **96**, 270 (1989) [Sov. Phys. JETP **69**, 152 (1989)].
[5] K. Walasek and K. Lukerska-Walasek, Phys. Rev. B **38**, 725 (1988).
[6] Ma Yu-giang, Li Zhen-ya, Phys. Lett. A **148**, 134 (1990).
[7] R. Pirc, B. Tadic, and R. Blinc, Z. Phys. B **61**, 69 (1985).
[8] H. T. Höchli, H. E. Weibel, and L. A. Boatner, J. Phys. C **12**, L563 (1979).
[9] M. Saint-Paul and J. G. Cilchrist, J. Phys. C **19**, 2091 (1986).
[10] M. C. Foote and B. Golding, J. Phys. Cond. Matt. **1**, 7751 (1989).
[11] S. Miyako, Chikazawa, and T. Saito, J. Phys. Soc. Jpn. **46**, 1951 (1979).
[12] K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).
[13] G. Williams, Can. J. Phys. **65**, 1251 (1987).
[14] S. Kirkpatrick and D. Sherrington, Phys. Rev. B **17**, 4384 (1978).
[15] K. Wada and H. Takayama, Prog. Theor. Phys. **64**, 327 (1980).

Translated by D. Parsons