Anisotropy of the upper critical field in exotic superconductors

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While the superconducting order parameter is characterized by a multidimensional representation of the rotation group (a state of the type ^{3}He-A), the upper critical field near T_c is anisotropic even in a crystal with a high degree of symmetry. This circumstance can be used to describe the nature of the observed superconductivity. The degeneracy can in principle be lifted by elastic stress.

Recent experiments[1] on the measurement of the specific heat in the superconductor UBe_{13} at low temperatures provided the first direct proof that an analog of the superfluid A phase of ^{3}He is realized in this compound. The dependence $C_e \sim T^3$ (instead of the usual activation dependence) indicates the presence of zeroes in the order parameter on the Fermi surface, just as in the Anderson–Morell–Brinkman–Rice state. It has already been pointed out (see, for example, Ref. 2) that an unusual superconductivity can exist in several compounds with the so-called "heavy fermions" ($CeSi_2Cu_2$[3], U_6Fe[4], UPt_3[5] and others), which have many exotic properties.

The mechanisms leading to nontrivial pairing are not discussed below. In the present letter we show that in the case of such pairing one of the main characteristics of superconductivity—the upper critical field H_{c2}—exhibits a considerable anisotropy near T_c even in a crystal with a high degree of symmetry (a cubic symmetry, for

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example). This anisotropy distinguishes the observed superconductivity from the usual superconductivity and imposes some restrictions on the ground-state symmetry. This is important, since the presence of zeroes in the energy gap does not determine this symmetry uniquely.[6]

Near the transition point the Ginzburg–Landau expansion is applicable for an anisotropic metal.[7,8] For the problem of the upper critical field H_{c2}, it is sufficient to restrict the analysis to only the gradient terms in the equation for the order parameter $\hat{\Delta}$ (**p**). The transition point is determined by a condition of the form

$$\hat{\Delta}(\mathbf{p}) = \ln \frac{\bar{\omega}}{T_c} \int \hat{K}(\mathbf{p}, \mathbf{p}') \hat{\Delta}(\mathbf{p}') d\Omega_{\mathbf{p}'} . \tag{1}$$

In (1) $\hat{\Delta}$ and the interaction $\hat{K}$ depend on spinor indices. By slightly changing the normalization $\hat{\Delta}$, we can put the kernel $\hat{K}(\mathbf{p},\mathbf{p}')$ in a Hermitian form. This kernel is invariant under all transformations of the crystal-symmetry group and under time reversal. It is appropriate to note here the spin nature of the order parameter.

The systems under discussion contain atoms of heavy elements, and the spin-orbit interaction in them is large. If we assume that the order parameter can be classified by assigning parity to spatial inversion, we will have two alternatives[9]:

$$\begin{aligned} \hat{\Delta}(\mathbf{p}) &= i\hat{\sigma}_y \psi(\mathbf{p}); \qquad (S=0), \\ \hat{\Delta}(\mathbf{p}) &= (\hat{\sigma}\mathbf{d}(\mathbf{p}))i\hat{\sigma}_y; \qquad (S=1). \end{aligned} \tag{2}$$

Substituting them separately into (1), we note that in the first case we obtain a scalar integral equation for $\psi(\mathbf{p})$, while in the second case we will obtain a vector equation for $\mathbf{d}(\mathbf{p})$. The strong spin-orbit coupling (the spins are "frozen" into the lattice and rotate together with it under the symmetry transformations) accounts for the fact that the solutions of Eq. (1) in both cases realize the irreducible representations of a subgroup of rotations of the corresponding complete point group of the crystal. As far as the basis functions are concerned, the complete set of basis functions for $\psi(\mathbf{p})$ is described, for UBe_{13}, for example, by even representations of the group O_h, while the vector basis functions of the corresponding representations for $\mathbf{d}(\mathbf{p})$ are constructed from the direct products of the spin unit vectors $(\tilde{x}, \tilde{y}, \tilde{z})$ and odd scalar representations of O_h. Since $S=0$ and $S=1$ can, according to (2), be decoupled, it is sufficient that the corresponding Ginzburg–Landau functional be invariant with respect to the rotation groups. Denoting the basis functions of the irreducible representation by ϕ^i, we can write the order parameter in each case in (2) as

$$\hat{\Delta}(\mathbf{p}) = \Sigma \, \eta_i \phi^i(\mathbf{p}) . \tag{3}$$

The group O has five representations (in the notation of Ref. 10): two one-dimensional representations, A_1 and A_2; one two-dimensional representation, E; and two three-dimensional representations, F_1 and F_2. As usual, the coordinate transformations are assumed to operate on the coefficients η_i and not on $\phi^i(\mathbf{p})$.

In selecting a unitary representation for the order parameter, the anisotropy of the crystal is taken into account near T_c only by the mass tensor.[7,8] The field H_{c2} near

T_c is thus isotropic in a cubic crystal and independent of the angle in the plane perpendicular to the principal axis for the tetragonal or hexagonal symmetry. For the one-dimensional (nonunitary) representation of any group, the problem, of course, reduces to the mass tensor (see the derivation in Ref. 8).

We now consider the degenerate solutions of (1) in the cubic group. In the two-dimensional representation E, we can select the basis functions in such a way that they would transform as

$$\phi^1 = x^2 + \epsilon y^2 + \epsilon^{-1} z^2 ; \quad \phi^2 = \phi^{1*} ; \quad \epsilon = \exp(2\pi i/3) . \tag{4}$$

The product of gradient-invariant derivatives $\partial_i \partial_k$, ($\partial_i = \partial/\partial x_i - ieA_i/c$) appears in the free-energy functional only with $i = k$. It is convenient to write the following combinations:

$$\Delta = \partial_{ii}^2 ; \quad \Delta_1 = \partial_{xx}^2 + \epsilon \partial_{yy}^2 + \epsilon^{-1} \partial_{zz}^2 ; \quad \Delta_2 = \partial_{xx}^2 + \epsilon^{-1} \partial_{yy}^2 + \epsilon \partial_{zz}^2 . \tag{5}$$

The gradient terms in the functional for the E representation are

$$\frac{1}{2m} (\eta_1^* \Delta \eta_1 + \eta_2^* \Delta \eta_2) + \frac{1}{2m'} (\eta_1^* \Delta_2 \eta_2 + \eta_2^* \Delta_1 \eta_1). \tag{6}$$

The basis functions of the three-dimensional representation F_2 transform like the symmetric tensor $\xi_{ij} = (xy, yz, zx)$, i.e., with $i = j$ $\xi_{ii} = 0$. A classification based on the irreducible representations shows that there are three invariants, which can be chosen, for example, as follows:

$$\xi_{ji}^* \partial_{il}^2 \xi_{lj} ; \quad \xi_{ij}^* \partial_{ll}^2 \xi_{ji} ; \quad (\xi_{xy}^* \partial_{yz}^2 \xi_{zx} + \ldots). \tag{7}$$

In the vector basis $\eta = (\eta_x, \eta_y, \eta_z)$ of the representation F_1 we have the following cubic invariants:

$$\eta_i^* \partial_{jj}^2 \eta_i ; \quad \eta_i^* \partial_{ij}^2 \eta_j ; \quad \eta_i^* \partial_{ii}^2 \eta_i . \tag{8}$$

The linear equation of stability, which determines H_{c2}, can be obtained through appropriate variation of the functional and in all cases (5)–(8) leads to an unexpected (for a cubic group) anisotropy of the upper field, which should be sought near T_c in UBe_{13}. Analysis of the angular dependence of H_{c2} in cubic and hexagonal structures is rather complicated and will be carried out separately. The simplest manifestation of the effect is in tetragonal symmetry (D_4), to which the $CeSi_2Cu_2$ and U_6Fe superconductors belong.

The group D_4 has only one two-dimensional representation (η_x, η_y), and the equations determining the field H_{c2} can be written [compare with (8)]

$$\frac{1}{2m_1} \partial_{ii}^2 \eta_x + \frac{1}{2m_2} \partial_{zz}^2 \eta_x + \frac{1}{2m_3} \partial_{xy}^2 \eta_y = (T - T_c)\eta_x \tag{9}$$

(in a corresponding way, they can be written for η_y). After selecting the field in the basal plane ($H\cos\varphi$, $H\sin\varphi$), and after simple calculations, we find from (9)

$$H_{c2}^{\perp}(\varphi) \propto (m_1 m_2)^{1/2} \left(1 - \frac{m_1}{m_3} |\sin\varphi \cos\varphi| \right)^{-1/2} . \tag{10}$$

Observation of the aforementioned singularities in the anisotropy of the field H_{c2} is thus a convenient method for determining the degenerate representation of the order parameter in (1) (the spin of the pair, however, is not determined). We recall that most high-symmetry classes, which have points or lines where the gap vanishes, are associated precisely with such representations.[6] Acoustic deformations, we might note, also lift the degeneracy of the order parameter, which leads to a splitting of the transition temperature in (1) by the elastic field. The effect, however, has a small value T_c/T_F because of the electron-hole symmetry near the Fermi level.

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