Phase-transition splitting caused in exotic superconductors by impurities

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The observed splitting of the superconducting transition in $U_{1-x}Th_xBe_{13}$ is explained on the basis that with increasing temperature the thorium impurity initially disrupts the orientational long-range order characteristic of exotic superconductors {G. E. Volovik and L. P. Gor'kov, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 550 (1984) [JETP Lett. **39**, 550 (1984)]}. The superconducting state is destroyed during a second transition.

Several experiments have shown that "heavy-fermion" systems (UBe_{13}, UPt_3) fall in a class of superconductors different from that of ordinary superconductors (the symmetry principle for classifying superconductors on the basis of the type of superconductivity is given in Ref. 1). The power-law, rather than exponential, temperature dependence of quantities such as the specific heat,[2] the ultrasonic damping,[3] and the spin-relaxation time indicates that there are zeros in the superconducting gap. This situation is characteristic of superconductors from nontrivial superconductivity classes, in which the gauge symmetry and spatial symmetry are coupled in a nontrivial way. A splitting of the phase transition caused by a thorium dopant, which replaces uranium, has been observed in UBe_{13} (Ref. 4). We will show below that this splitting is a natural consequence of the symmetry of a superconductor from a nontrivial class of superconductivity. We will discuss here only one possible splitting mechanism, involving a disruption of the orientational long-range order. We will describe it by means of a Ginzburg-Landau functional.

The order parameter in a superconductor is the set of coefficients in the expansion of the binary amplitudes $\langle a_{\mathbf{k}\alpha} a_{-\mathbf{k}\beta} \rangle$ in terms of the basis functions of an irreducible representation of the point symmetry group of crystal[5]; for UBe_{13}, this is the cubic group O_h. We consider representations with a maximum dimensionality of 3 in the O_h group, since the mechanism being described does not cause a splitting at a lower dimensionality. The O_h group has four three-dimensional representations: the even representations F_{1g} and F_{2g}, which correspond to a singlet pairing, and the odd representations F_{1u} and F_{2u}, which correspond to a triplet pairing. In all of these representations, three of the coefficients of the basis functions can be chosen in the form of a complex vector $\vec{\psi}$, in terms of which the Ginzburg-Landau functional, invariant under the group O_h, has the following general form:

$$F = -\alpha\left(1 - \frac{T}{T_c}\right)|\vec{\psi}|^2 + \beta_1 (|\vec{\psi}|^2)^2 + \beta_2 |\vec{\psi}^2|^2 + \beta_2 (|\psi_x|^4 + |\psi_y|^4 + |\psi_z|^4)$$
$$+ K_1 |\partial_i \vec{\psi}|^2 + K_2 \partial_i \psi_i \partial_k \psi_k^* + K_3 \partial_i \psi_k \partial_k \psi_i^* \qquad (1)$$
$$+ K_4 (|\partial_x \psi_x|^2 + |\partial_y \psi_y|^2 + |\partial_z \psi_z|^2).$$

The cubic crystalline anisotropy is represented by the terms with β_3 and K_4.

The minimum of the Ginzburg-Landau functional occurs in different states (in different superconducting phases), depending on the relations among β_1, β_2, and β_3. Any of these phases is degenerate under the group O, demonstrating a long-range orientational order in these phases. For example, a solution in the form of a real vector $\vec{\psi} \sim (1, 1, 1)$ implies the appearance of a spontaneous anisotropy axis along one of the threefold axes; such a state is quadruply degenerate. A sixfold-degenerate solution in the form of a complex vector $\vec{\psi} \sim (1, i, 0)$ implies a spontaneous magnetic moment along one of the fourfold axes (the z axis).

Impurities at the points $\mathbf{r}_a$ add to the Ginzburg-Landau functional a term quadratic in $\vec{\psi}$:

$$F_{\rm im} = \sum_a \psi_i \psi_k^* U_{ik}^a \delta(\mathbf{r} - \mathbf{r}_a). \qquad (2)$$

This term describes the interaction of the impurities with the order parameter. In contrast with an ordinary superconductor, in which this interaction would be described by a scalar potential, the matrix U_{ik}^a is anisotropic in an exotic superconductor and depends on the position of the impurity a with respect to the center of the unit cell. As a result, the impurity not only shifts T_c by an amount equal to the average potential,

$$\Delta T_c \sim T_c n < U > / \alpha \sim \frac{1}{\tau} \qquad (3)$$

(n is the impurity density, and τ is the scale transit time of the carriers), but also creates a random local anisotropy for $\vec{\psi}$.

In the absence of a regular crystalline anisotropy ($\beta_3 = 0$), when there is a continuous degeneracy in terms of the orientation of $\vec{\psi}$, a local random anisotropy disrupts the long-range orientational order in three-dimensional space.[6] The orientation of the order parameter becomes irregular, varying over a scale distance L substantially greater than the average distance between impurities, R. The inhomogeneity scale length L is found by comparing the energy gain due to the orientation of the order parameter by the fluctuational field of the impurities, which is proportional to the square root of the number of impurities in a region of size L $(\int dV(F_{\rm im} - \langle F_{\rm im}\rangle) \sim (L/R)^{3/2}(\langle U^2\rangle - \langle U\rangle^2)^{1/2}|\vec{\psi}|^2)$, with the loss in the gradient energy, $\sim KL\,|\vec{\psi}|^2$, due to the formation of an inhomogeneous state:

$$L \sim \frac{K}{n\,(<U^2>-<U>^2)}\,. \tag{4}$$

If $\beta_3 \neq 0$, the loss of orientational order occurs only if the gain in fluctuational energy exceeds not only the loss in radiant energy but also the loss in the energy of the regular crystalline anisotropy, $\sim\beta_3|\vec{\psi}|^4L^3$. For this condition to hold, the scale dimension of the inhomogeneity in (4) must be smaller than $\xi\,(\beta_1/\beta_3)^{1/2}$, where $\xi = \xi_0(1 - T/T_c)^{-1/2}$ is the coherence length ($\xi_0^2 \sim K/\alpha$). This condition on the loss of orientational order holds near T_c in the temperature interval

$$1 - \frac{T}{T_c} < \left(\frac{R}{\xi_0}\right)^6 \frac{1}{(\tau T_c)^4}\,\frac{\beta_1}{\beta_3}\,. \tag{5}$$

The length L may still be greater than ξ, so that the superconductivity is not yet destroyed. Consequently, near T_c there should be a superconducting state in which the long-range orientational order is disrupted by a random impurity anisotropy. Such a state would naturally be called a "superconducting glass." The order parameter in it is the complex scalar $\langle\vec{\psi}\vec{\psi}\rangle$.

Figure 1 is a sketch of the phase diagram. The dotted curve corresponds to the suppression of T_c due to the isotropic part of the impurity potential. This effect is completely analogous to the suppression of T_c by paramagnetic impurities in ordinary superconductors.[7] The splitting of the transition into orientational and superconducting transitions, δT_c, increases in proportion to x^2 at low concentrations x and then approaches zero as x approaches the critical concentration, where the transition temperature T_c approaches zero: Noting that $\delta_0 \sim T_c^{-1}$, we find from (5) that we should

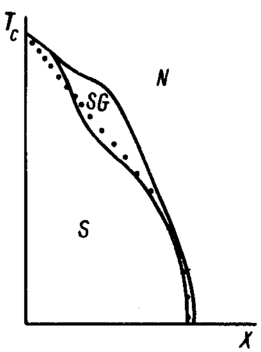

FIG. 1. Phase diagram for an exotic superconductor containing an impurity in a concentration x. The solid lines show the splitting of the superconducting transition against the background of a general suppression of the transition temperature (the dotted line). N—Normal metal; S—superconducting phase with an orientational long-range order; SG—superconducting phase with a disrupted orientational order (a "superconducting glass").

have $\delta T_c \sim T_c^3$ as $T_c \to 0$. Furthermore, if $\xi_0 > R$, the approach of δT_c to zero is even more rapid, since the anisotropic impurity potentials in this case are weakened by a mutual cancellation. Effectively, the splitting should thus be apparent only at intermediate concentrations, in agreement with the experimental data on $U_{1-x}Th_xBe_{13}$, where the splitting is observed[4] at $0.02 < x < 0.04$.

If we assume that this type of orientational transition occurs in UBe_{13}, then we can draw a conclusion about the dimensionality of the representation. Since the local anisotropy is related to a splitting of the representation due to an impurity that disrupts the cubic symmetry, this representation must have more than one dimension. Furthermore, if the local anisotropy axis coincides with a threefold axis, the representation cannot be two-dimensional, since the latter would not split upon such a disruption of symmetry. Consequently, in UBe_{13}, in which the Th atoms replacing the U atoms lie on threefold axes, the superconducting state is characterized by a three-dimensional representation. This representation is probably one of the even representations F_{1g} or F_{2g} corresponding to singlet pairing, since it is for these representations that superconducting phases with gap-zero lines exist; for the triplet phases, the zeros lie at points on the Fermi surface. Recent experiments[3,4] imply that the gap zeros lie on lines of the Fermi surface.

Other mechanisms for the splitting of the superconducting transition are also possible; examples are mechanisms involving changes in the relations among the parameters β_1, β_2, and β_3 in the Ginzburg-Landau functional upon a change in x. In any case, the existence of a splitting indicates that the irreducible representation which is responsible for the superconducting transition is multidimensional.

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