Conductance anomaly at superconductor–semiconductor contacts

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A microscopic theory is derived for *SIN'IN* contacts whose length is greater than the correlation length. Boundary conditions are found for the distribution functions at the *SIN* boundary. Expressions are found for the conductance peak at a zero bias voltage in *SIN'IN* contacts and for the dependence of this peak on the magnetic field.

Contacts of various types have recently attracted increased interest. In particular, measurements have been carried out on *S–Sm* contacts, which behave like *SIN* contacts (*Sm*, *S*, *N*, and *I* are respectively a semiconductor, a superconductor, a normal metal, and an insulating layer).[1,2] The role of insultating layer *I* is played by the Schottky barrier which arises at the boundary of the superconductor and the heavily doped semiconductor. A peak was observed in the differential conductance of Nb–$n^+In_{0.53}Ga_{0.47}As$ contacts at a zero bias voltage at low temperatures ($T<1$ K) in Ref. 1. That peak could be suppressed by a weak field ($H<100$ mT). Kastalsky *et al.*[1] attributed this peak to a prixirnity effect, i.e., to the appearance of superconducting correlations in the *Sm* region. For a quantitative interpretation, they used equations derived by Geshkenbeĭn and Sokol[3] for an *SIN* contact on the basis of the nonequilibrium Ginzburg–Landau equations. They generalized those equations, taking the magnetic field into account.

The application of the microscopic theory to ordinary (i.e., not gapless) superconductors has been held up by the lack of boundary conditions for the Green's functions at the *SIN* boundary. The Josephson effect in the *SINIS* system was apparently first studied by Aslamazov and Ovchinnikov.[12] They joined the solutions at the *SIN* boundary by a tunneling-Hamiltonian method. Joining conditions (valid for any transmittance) for the Green's functions at an *SIN* boundary were recently found without the help of that method by Zaĭtev[4] (for the general case) and by Kupriyanov and Lukichev[5] (for the dirty case). Using these conditons, Zaĭtsev[6] analyzed the current-voltage characteristics of dirty *SININ* and *SINIS* contacts with a length shorter than the correlation length ξ. He found by numerical calculations that the conductance has a peak at $V=0$ in *SIN'IN* contacts with a low transmittance. He also found from an excess current $I_{exc}>0$ to a deficient current $I_{def}<0$ with decreasing transmittance of the barrier. A transition of this sort had been found previously by Blonder *et al.*[7] on the basis of a simpler and clearer theory, but that theory did not predict a conductance anomaly at a zero bias voltage, since the proximity effect was disregarded.

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In the present letter we derive joining condition for the distribution functions at an *SIN* boundary by working from the boundary conditions for Green's functions.[5] These conditions make it possible to analyze contacts with a length $d > \xi$. Using them, we study dirty *SIN'IN* contacts with a length d which satisfies the condition $\xi_N < d < l_\epsilon$, where ξ_N is the superconducting correlation length in the N' region (this length is assumed to be large in comparison with the mean free path l), and $l_\epsilon = (D_N \tau_\epsilon)^{1/2}$ is the energy relaxation length. The latter condition allows us to ignore the inelastic-collision integral in the N' region. We derive analytic equations describing the current–voltage characteristics of an *SIN'IN* contact. In particular, we find the shape of the conductance peak at $V = 0$ and its dependence on the magnetic field.

The equation for the isotropic part of the Green's-function matrix $\check{G}$ satisfies the following equation[8] in the N' region:

$$D_N \partial_x (\check{G} \partial_x \check{G}) + i\epsilon[\check{\sigma}_z, \check{G}]_- = 0. \tag{1}$$

Here $\check{G}$ is a supermatrix whose elements are the functions $\hat{G}^{R(A)}$ and $\hat{G} = \hat{G}^R \hat{f} - \hat{f} \hat{G}^A$, where $\hat{f} = f_z \hat{\sigma}_z + f_1 \hat{1}$ is the matrix of distribution functions. The function f_z, in which we are interested, determines the electric current and the potential.[8,9] We impose on $\check{G}$ the boundary conditions found in Ref. 5 for an *SIN* barrier, taking account of the difference between the Fermi momenta to the right and left of the barrier:

$$r_0 l \, \check{G} \partial_x \check{G} \, |_{x=0} = [\check{G}_S, \check{G}_0]_-. \tag{2}$$

The coefficient r_0 characterizes the transmittance of the *SIN* boundary. The boundary resistance R_b is expressed in terms of r_0: $R_b = r_0 l / 2\sigma_N$, where σ_N is the conductivity in the N' region. The x axis runs perpendicular to the interface which is at $x = 0$. Writing retarded Green's function $\hat{G}^R$ in the form $\hat{G}^R = \hat{\sigma}_z \cosh u^R + i\hat{\sigma}_y \sinh u^R$, we can solve the equation for $\hat{G}^R$ exactly in the N' region. The solution, with the boundary condition $\hat{G}^R(\infty) = \hat{\sigma}_z$, is

$$\tanh(u^R(x)/4) = \tanh(u_0^R/4) \exp(-k^R x). \tag{3}$$

A corresponding equation can be derived for $\delta u^A \equiv u^A - i\pi$. The constants u_0^R and δu_0^A are found from conditions (2) at $x = 0$. They satisfy the equations

$$(r_0 l k^R) \sinh(u_0^R/2) = f^R \cosh u_0^R - g^R \sinh u_0^R,$$
$$(r_0 l k^A) \sinh(\delta u_0^A/2) = -f^A \cosh \delta u_0^A + g^A \sinh \delta u_0^A. \tag{4}$$

Here $k^{A(R)} = [\pm 2i\epsilon/D_N]^{1/2}$ and g^R and f^R are the components of $\hat{G}_S^R$ in the S region, which are assumed to be unperturbed:[1)] $g^{R(A)} = \epsilon/\xi^{R(A)}$, $f^{R(A)} = \Delta/\xi^{R(A)}$, $\xi^{R(A)} = [(\epsilon \pm i0)^2 - \Delta^2]^{1/2}$.

To find the joining conditions for f_z, we need to calculate element (12) of Eq. (1), multiply it by $\hat{\sigma}$, and calculate the trace. We then find that the flux

$$l(\partial_x f_z)[1 - \cosh(u^R + u^A)] \equiv I_z(\epsilon) \tag{5}$$

is independent of x, and the function f_z itself varies only slightly over distances on the order of ξ_N. For f_z we have[10]

$$f_z(x) = f_z(0) + I(\epsilon)(x/2l). \tag{6}$$

From Eqs. (2) we find boundary condition on f_z at the SIN boundary:

$$r_0 I_z(\epsilon) = A_0[f_z(0) - f_z^S]. \tag{7}$$

Here $A_0 = (g^R - g^A)(G_0^R - G_0^A) - (f^R + f^A)(F_0^R + F_0^A)$, and f_z^S is the distribution function in the S region, which is zero at equilibrium (if the phase is chosen to be zero). The functions $G_0^R = \cosh u^R$ and $F_0^R = \sinh u^R$ are found from (4). We can thus work from Eqs. (4), (6), and (7) [the joining condition at $x = d$ is of the same form as (7), with $A_0 = 4$] to calculate the current in the system:

$$j = (\sigma_N/4l)\int d\epsilon I_z(\epsilon). \tag{8}$$

Let us consider an $SIN'IN$ contact. From Eqs. (6) and (7) and from the boundary condition at $x = d$, with $A_0 = 4$, we find

$$I(\epsilon) = \frac{4F_-}{r_d + r_0(4/A_0) + 2d/l}, \tag{9}$$

where $F_- = [\tanh(\epsilon + V)\beta - \tanh(\epsilon - V)\beta]/2$ is the equilibrium distribution function in the N electrode when there is a voltage V. Using (8) and (9), we can find the current–voltage characteristic of the contact. Equations (4), which determine $A_0(\epsilon)$, can be solved analytically in the cases of a high transmittance ($r_0 \ll \xi_N/l$) and a low transmittance ($r_0 \gg \xi_N/l$) of the SIN' barrier.[10] At absolute zero, the reduced differential conductivity $\tilde{\sigma}_d = R_N \partial j/\partial V$ is $\tilde{\sigma}_d(v) = (1+a)/[a + 4/A_0(v)]$, where $a = (2d + r_d l)/r_0 l$, and $R_N = [d + (r_d + r_0)l/2]\sigma_N$ is the resistance of the contact in its normal state. In the case of a low transmittance we find the following expression for $A_0(\epsilon)$:

$$A_0(v)/4 = \frac{|v|}{(v^2 - \Delta^2)^{1/2}}\theta(|v| - \Delta) + \frac{\Delta}{(\Delta^2 - v^2)}[1 + \tilde{r}_0\sqrt{|v|/\Delta}]^{-1}\theta(\Delta - |v|), \tag{10}$$

where $\tilde{r}_0 \equiv r_0 l/\sqrt{2}\xi_N(\Delta) \gg 1$, $\xi_N(\Delta) = (D_N/2\Delta)^{1/2}$, and $v = eV/\Delta$. It is not difficult to see that under the conditions $\xi_N(\Delta)/d \ll \sqrt{v} \ll 1$ we have $\tilde{\sigma}_d(v) = [1 + \sqrt{v/v_0}]^1$, where $v_0 = [(1+a)/\tilde{r}_0]^2$ is a characteristic voltage for the conductance anomaly at a zero bias voltage. It can also be shown that in contacts with a high transmittance an excess current $I_{\rm exc}$ arises ($I_{\rm exc} R_N = 4\Delta/3a$ at $a \gg 1$), while in contacts with a low transmittance a deficient current $I_{\rm def}$ arises [$I_{\rm def} R_N = -(8/21)\tilde{r}_0/a$ with $a \gg \tilde{r}_0 \gg 1$]. They decrease with increasing length of the contact.[10]

The imposition of a magnetic field H (parallel to the z axis) accounts for the presence of a phase in the S region: $\chi(y) = 2\pi\lambda y H/\Phi_0$, where λ is the London depth. The Green's functions in the presence of a phase, $\hat{G}_\chi^{R(A)}$, can be expressed in terms of the functions in the absence of a phase by means of a transformation $\hat{G}_\chi^R = \hat{S}(y)\hat{G}^R\hat{S}^+(y)$, where $\hat{S}(y) = \cos[\chi(y)/2] + i\hat{\sigma}_z \sin[\chi(y)/2]$. The equation for u^R then changes, as does the solution of this equation [see Eq. (3)].[11] Because of this change, the left side of Eqs. (4) does not vanish as $\epsilon \to 0$, since $k^R(\epsilon)$ is replaced by $[(k^R)^2 + (\partial_y \chi)^2]^{1/2}$. Omitting the details of the calculations, we write the result for $\tilde{\sigma}_d(v)$ for $\xi_N(\Delta)/d \ll \sqrt{v} \ll 1$ and $T = 0$:

$$\tilde{\sigma}_d(v) = [1 + [(h^4 + v^2)/(v_0(h^2 + (h^4 + v^2)^{1/2}))]^{1/2}]^{-1}. \tag{11}$$

Here $h = 2\pi\lambda\xi_N(\Delta)H/\Phi_0$. For $v \ll v_0$ we find the dependence of $\tilde{\sigma}_d(0)$ on H from (11): $\tilde{\sigma}_d(0) = 1/[1 + |H|/H_0]$, where $H_0 = \Phi_0\sqrt{2v_0}/[2\pi\lambda\xi_N(\Delta)]$ is a characteristic magnetic field at which the anomaly is suppressed. This is the dependence which was observed in Ref. 1. At nonzero temperatures, the $\tilde{\sigma}_d(0)$ peak is suppressed. The characteristic temperature of this suppression is $T_0 \simeq \Delta v_0$.

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[1)]Because of the proximity effect, the functions $g^{R(A)}$ also change in the S region. This change is small and can be ignored if the barrier transmittance is small or if N' is a narrow constriction.

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