Sign reversal of the flux-flow Hall effect in type-II superconductors

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For BCS model of superconductivity, the Hall voltage in the mixed state is shown to be proportional to the energy derivative of the quasiparticle density of states. The sign of the Hall effect in the mixed state may turn out to be different from that in the normal state. In the latter case, the Hall angle changes its sign as a function of the magnetic field below H_{c2}.

INTRODUCTION

A sign reversal of the Hall angle as a function of magnetic field has been observed in recent experiments on high-temperature superconductors,[1–4] and several theoretical models[1,3,5,6] have been suggested to explain this behavior. The change of sign, however, is not a new phonomenon for the Hall effect in the mixed state: it has been observed for V and Nb already in the 1970s (see Ref. 7 and the bibliography cited there). This effect seems to be quite general and needs an explanation in terms of the vortex dynamics not specific for high-temperature superconductors alone. Experimental conditions for the Hall-effect measurements in superconductors correspond to the low-field limit since the cyclotron frequency is $\omega_c \ll \tau^{-1}$ when H does not exceed H_{c2}. The normal-state Hall conductivity is $\sigma_n^H \sim (\omega_c \tau)\sigma_n$ and is small compared to the usual ohmic part σ_n. This implies that the Hall effect in a not very pure superconductor can be treated as a perturbation against the background of a viscous flow of vortices. (In Refs. 8 and 9 the Hall effect was considered microscopically for the opposite, extremely pure limit.)

In the present paper we derive a time-dependent Ginzburg–Landau theory which includes two mechanisms which are responsible for the Hall voltage. The first mechanism is the usual effect of the magnetic field on the normal current. The second mechanism, peculiar to vortices, is their traction by the superflow: the vortices have a velocity component parallel to the transport supercurrent. This gives rise to a Hall voltage since the averaged electric field is perpendicular to the vortex velocity $\vec{v}_L$. The second mechanism dominates in fields below H_{c2}.

MODIFIED TDGL EQUATIONS

The time-dependent Ginzburg–Landau (TDGL) equations are

$$-\gamma\left(\frac{\partial \Psi}{\partial t} + 2ie\phi\Psi\right) = \frac{\delta \mathcal{F}_{sn}}{\delta \Psi^*} \quad ; \tag{1}$$

$$-\frac{1}{c}(\vec{j} - \vec{j}_n) = \frac{\delta \mathcal{F}_{sn}}{\delta \vec{A}} \quad . \tag{2}$$

Here $\mathscr{F}_{sn}$ is the condensation free energy of a superconductor,

$$\mathcal{F}_{sn} = \int [C_1 |\Psi|^2 + \frac{1}{2} C_2 |\Psi|^4 + \frac{1}{2m} |(-i\vec{\nabla} - \frac{2e}{c}\vec{A})\Psi|^2] dV \ . \qquad (3)$$

To account for the Hall effect within the TDGL formalism, we should include the Hall component in the normal current:[10,11]

$$\vec{j}_n = \sigma_n \vec{E} + \sigma_n^H [\vec{E} \times \vec{H}]/H \ . \qquad (4)$$

This contribution, however, is not the only one. In the dissipative flux-flow regime, vortices move perpendicular to the transport current, so that the average electric field induced in the superconductor is parallel to the transport current. In the ideal fluid, however, a vortex moves together with the flow; this complete vortex traction by the flow is a consequence of the Galilean invariance. For the TDGL model, there is no Galilean invariance since the excitations are at rest with the crystal lattice. However, some vortex traction can still exist: it can appear through a small, imaginary part of the relaxation constant $\gamma = \gamma' + i\gamma''$ in Eq. (1). If γ were purely imaginary, Eq. (1) would be a (nonlinear) Schrödinger equation, which is a Galilean invariant.

The TDGL model with a complex γ can be justified by the microscopic theory of nonstationary superconductivity. To account for the Hall effect, we must go beyond the quasiclassical approximation generally used in the theory of superconductivity and take into account the energy dependences of such quantities as the density of states, the pairing potential (the phonon Green's function for the phonon model of superconductivity, for example), relaxation times, etc. In the present paper we consider the simplest example of the BCS model of superconductivity. In this model the only contribution to the vortex traction is due to the energy dependence of the quasiparticle density of states.

The TDGL equations can be derived only for gapless superconductors. We consider a simple case of a gapless regime for a weak pair breaking with the characteristic time τ_0 such that $\tau_0 T_c \gg 1$. The ratio of the imaginary part to the real part of γ is

$$\zeta \equiv -\frac{\gamma''}{\gamma'} = \frac{4T}{\pi\nu(0)} \left[\frac{\partial \nu(0)}{\partial \xi_p}\right] \left(\frac{1+\lambda}{\lambda}\right), \qquad (5)$$

where λ is the BCS pairing constant.

The imaginary part γ'' is proportional to the derivative of the density of states $\nu(\xi_p)$ with respect to the quasiparticle energy $\xi_p = \epsilon_p - E_F$, taken at the Fermi surface, i.e., for $\xi_p = 0$. It is on the order of T_c/E_F. It is nevertheless very important for the Hall effect in the mixed state of superconductors.

VORTEX MOTION AND THE HALL EFFECT

Low-magnetic fields, $B \ll H_{c2}$

The variation of the total free energy, i.e., the condensation energy, Eq. (3), together with the magnetic energy, caused by the displacement of the vortex lattice by an arbitrary vector $\vec{d}$, is[12] (omitting the surface terms)

$$\delta\mathcal{F} = \int \left((\vec{d}\vec{\nabla})\Psi \frac{\delta\mathcal{F}_{sn}}{\delta\Psi} + \text{c.c.} + (\vec{d}\vec{\nabla})\vec{A}\frac{\delta\mathcal{F}_{sn}}{\delta\vec{A}} + \frac{1}{4\pi}\vec{H}\ \text{curl}[(\vec{d}\vec{\nabla})\vec{A}] \right) dV \ . \qquad (6)$$

The free-energy variation of Eq. (6) is the work done by the force exerted by excitations. If the integration in Eq. (6) is extended over the area S_0 of one vortex-lattice unit cell, we obtain the force acting on one vortex. This force should be balanced by the external Lorentz force from the transport supercurrent. Therefore,[12]

$$\frac{\phi_o}{c}(\vec{d}[\vec{j}_{tr} \times \vec{n}]) = \int_{S_0} \left(-\gamma(\vec{d}\vec{\nabla})\Psi^*(\frac{\partial\Psi}{\partial t} + 2ie\phi\Psi) - \text{c.c.} + \frac{\sigma_n}{c}[(\vec{d}\vec{\nabla})\vec{A}]\vec{j}_n \right) dS. \qquad (7)$$

Here $\vec{n}$ is the unit vector of the vortex circulation, and $\phi_0 = \pi c/2e$ is the flux quantum.

The scalar potential ϕ in Eq. (7) is proportional to v_L. It obeys the equation which can be obtained as follows. Note that

$$\frac{\delta\mathcal{F}}{\delta\chi} \equiv i\left(\Psi\frac{\delta\mathcal{F}}{\delta\Psi} - \Psi^*\frac{\delta\mathcal{F}}{\delta\Psi^*}\right) = -\frac{1}{2e}\text{div}\vec{j}_s \ , \qquad (8)$$

where χ is the phase of the order parameter. From $\text{div}\vec{j} = 0$ and Eq. (1) we have

$$\nabla^2\Phi - \frac{8e^2\gamma' \mid \Psi \mid^2}{\sigma_n}\Phi = -\frac{1}{c}\text{div}\frac{\partial\vec{Q}}{\partial t} + \frac{2e\gamma''}{\sigma_n}\frac{\partial \mid \Psi \mid^2}{\partial t} - \frac{4\pi\sigma_n^H}{\sigma_n^2 Hc}\left(\vec{E}\vec{j} + \frac{\partial}{\partial t}(\frac{H^2}{8\pi})\right). \qquad (9)$$

Here $\Phi = \phi + (1/2e)(\partial\chi/\partial t)$, and $\vec{Q} = \vec{A} - (c/2e)\vec{\nabla}\chi$. The last two terms on the right-hand side of Eq. (9) are associated with the Hall effect.

For a slow vortex motion, the time derivative in Eqs. (7) and (9) can be replaced with $-\vec{v}_L\vec{\nabla}$, which act on the variables that describe a static vortex. Therefore, Eq. (7) contains either the known functions or the function Φ, which can be found from Eq. (9). The boundary conditions for Φ are: (1) $\Phi = 0$ for large distances from the vortex, and (2) the scalar potential ϕ is finite at the vortex center.

We consider a superconductor with a large Ginzburg–Landau parameter κ. In this case we can ignore the term with the vector potential $\vec{A}$ in Eq. (7) and the normal-state Hall contributions to Eq. (9).

The order parameter is $|\Psi| = |\Psi_0|f$, where $|\Psi_0|$ is its equilibrium value. For a single vortex, f is a function of the distance from the vortex axis, ρ, the order-parameter phase is just the azimuthal angle $\chi = \varphi$, and $\vec{Q} = (0, -c/2e\rho, 0)$ in the cylindrical coordinate frame (ρ,φ,z) associated with the vortex.

We set

$$\Phi_0 = -\frac{v_{L\varphi}}{2e\xi}\mu_0(\rho) \ , \quad \Phi_1 = -\zeta\frac{v_{L\rho}}{2e\xi}\mu_1(\rho). \qquad (10)$$

Here ξ is the temperature-dependent coherence length. The function μ_0 satisfies the equation

$$\xi^2(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{1}{\rho^2})\mu_0 - uf^2\mu_0 = 0 \qquad (11)$$

with the boundary conditions[13] $\mu_0 = 0$ for $\rho \to \infty$ and $\mu_0 = \xi/\rho$ for $\rho \to 0$. Here

$u = 8e^2\gamma'\xi^2|\Psi_0|^2/\sigma_n$ is the numerical factor equal to the ratio squared of ξ and the electric-field penetration length. For a weak pair breaking, $\tau_0 T_c \gg 1$, the factor $u = 5.79$. For $\tau_0 T_c \ll 1$ (high concentration of magnetic impurities) $u = 12$. The term with Φ_0 was obtained previously for purely dissipative flux flow. The new term μ_1 satisfies the equation

$$\xi^2\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{1}{\rho^2}\right)\mu_1 - uf^2\mu_1 = -u\xi f\frac{df}{d\rho} \tag{12}$$

with the boundary conditions $\mu_1 = 0$ for both $\rho = 0$ and $\rho \to \infty$.

Collecting all the terms in Eq. (7), we obtain

$$\vec{j}_{tr} = \sigma_f \vec{E} + \sigma_f^H[\vec{E} \times \vec{H}]/H, \tag{13}$$

where

$$\sigma_f = \frac{\alpha u \sigma_n}{2}\left(\frac{H_{c2}}{B}\right), \quad \text{and} \quad \sigma_f^H = \text{sign}(e)\frac{\zeta\beta u\sigma_n}{2}\left(\frac{H_{c2}}{B}\right) \tag{14}$$

are the ohmic and the Hall conductivities in the flux-flow regime, respectively. The sign of the electron charge in σ_f^H appears since the circulation of the phase is chosen for the positive charge of the carriers.

The constants

$$\alpha = \int_0^\infty \left(\rho\left(\frac{df}{d\rho}\right)^2 + \frac{f^2\mu_0}{\xi}\right) d\rho, \tag{15}$$

and

$$\beta = \int_0^\infty \left(\frac{1}{2}\left(1 + \frac{\rho\mu_0}{\xi}\right)\frac{df^2}{d\rho} - \frac{f^2\mu_1}{\xi}\right) d\rho \tag{16}$$

can be calculated numerically from the solutions of Eqs. (11) and (12) using the known function f. The constant α was calculated previously (see Ref. 12): $\alpha \approx 0.502$ for $u = 5.79$, and $\alpha \approx 0.438$ for $u = 12$. Solving Eq. (12) for the function μ_1, we obtain $\beta \approx 0.27$ for $u = 5.79$. We can consider other values of u which model various pair-breaking mechanisms. For $u = 12$ (high concentration of magnetic impurities) $\beta \approx 0.21$. For small $u \ll 1$, we have $\beta = 1$, since $\mu_1 \sim u$ and $\mu_0 = \xi/\rho$.

High magnetic fields, $B \to H_{c2}$

In the limit of high magnetic fields, $H_{c2} - H \ll H_{c2}$, we need to solve the linearized TDGL equation with $\phi = -E_x x - E_y y$. Assuming $A_y = Bx$, $A_x = 0$, we find the solution within first-order terms in $\vec{E}$

$$\Psi = \sum_n C_n \exp[i(qn + 2eE_y t)(y + cE_x t/B)]$$

$$\times \exp\left[-\frac{1}{2\xi^2}\left(x - \frac{cE_y t}{B} - \frac{cqn}{2eB}\right)^2 + 2me\xi^2\gamma(iE_x - \text{sign}(e)E_y)\left(x - \frac{cqn}{2eB}\right)\right]. \tag{17}$$

This solution describes a slightly modified vortex lattice that moves with the velocity $\vec{v}_L = (cE_y/B;\ -cE_x/B)$. It is similar to that obtained in Ref. 14.

The order-parameter magnitude can be found from the nonlinear GL equation. The coefficients C_n correspond to the Abrikosov vortex lattice with the parameter $\beta_A \approx 1.16$. After calculating the average total current using Eqs. (4) and (17), we obtain Eq. (13) with

$$\sigma_f = \sigma_n + 4e^2\xi^2\gamma'\langle|\Psi|^2\rangle = \sigma_n\left[1 + \frac{u}{2}\frac{H_{c2}-B}{\beta_A H_{c2}}\right], \tag{18}$$

$$\sigma_f^H = \sigma_n^H + \text{sign}(e)\sigma_n\frac{\zeta u}{2}\frac{H_{c2}-B}{\beta_A H_{c2}}\Bigg]. \tag{19}$$

DISCUSSION

The ratio of the Hall and the flux-flow conductivities gives the Hall angle. For $B \ll H_{c2}$, the Hall angle is independent of the magnetic field: $\tan\Theta_H = \text{sign}(e)\beta\zeta/\alpha$. It is negative for quasiparticles with a positive derivative $(\partial\nu(0)/\partial\xi_p)$, and vice versa. The Hall angle becomes field-dependent as $B \to H_{c2}$:

$$\tan\Theta_H = \frac{\sigma_n^H}{\sigma_n} + \text{sign}(e)\zeta\frac{u}{2}\frac{H_{c2}-B}{\beta_A H_{c2}}. \tag{20}$$

In the normal state, $H > H_{c2}$, the sign of the Hall angle is controlled by the effective sign of the charge carriers, i.e., by the sign of σ_n^H. The Hall conductivity in the mixed state, however, contains the different quantity; i.e., γ'', which is proportional to the energy derivative of the density of states averaged over the Fermi surface. For a simple isotropic Fermi surface, the sign of the Hall conductivity in both the normal state and the mixed state is equal to the sign of (e/m^*). However, for a complicated Fermi surface, which has electron-like and hole-like parts, there is a new possibility: the signs of $(e\zeta)$ and σ_n^H can be different since they result from the averaging of the different sign-alternating quantities. If these signs are opposite, the Hall angle will change its sign with a decrease in the magnetic field below H_{c2}.

Both the same signs and the opposite signs of σ_f^H seem to be observed experimentally in the case of ordinary superconductors.[7,15] As a rule, the sign reversal is observed for pure Nb and V which have complicated Fermi surfaces, while there is no change in sign for dirty superconductors. According to our results, the sign reversal depends crucially on the shape of the Fermi surface. The effect of impurities might be a simplification of the Fermi surface due to the scattering. As a result, there would be no sign reversal. In addition, the shape of the Fermi surface depends on the position of the Fermi level. These may be the reasons why the experimental data for the Hall angle are so diverse for various samples.[7] Hall angle are so diverse for various samples.[7]

The experimental data for high-temperature superconductors are even more puzzling: The sign reversal is observed usually at temperatures near T_c and disappears at lower temperatures. This effect deserves further study. For example, one can investigate contributions to the imaginary part γ'', which can come from the energy dependences of the pairing potential and/or the scattering times in various models, including the phonon model of superconductivity. We will consider these effects elsewhere.

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