Sign and magnitude of the quadrupole interaction constant d of muonium in α quartz

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(Submitted 12 December 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 6, 278–280 (25 March 1984)

A novel method has been used to determine the sign of the quadrupole constant d of the muonium atom. The idea is to study the frequencies of transitions between the energy levels of triplet muonium ($F = 1$) as functions of the external magnetic field. At $T = 293$ K, the value $d = -0.517 \pm 0.008$ MHz is found.

As was shown in Ref. 1 and pursued theoretically in Refs. 2 and 3, the bound system of a positive muon, μ^+, and an electron (muonium, Mu) has a new quantum characteristic in its ground state: an electric quadrupole moment.

 0021-3640/84/060332-05$01.00

This moment splits the levels of the hyperfine structure when a Mu atom is placed in a nonuniform electric field (the field in the interior of a crystal, for example). In this case the spin Hamiltonian of muonium in the absence of an external magnetic field is

$$\hat{H} = A\mathbf{SJ} + \frac{1}{6} Q_{ik}\,\phi_{ik}\,. \qquad (1)$$

The first term in (1) is the ordinary hyperfine interaction between the spins of the muon (**J**) and the electron (**S**), with the hyperfine interaction constant A, while the second term is the quadrupole interaction with the nonuniform electric field. Here $\phi_{ik} = \partial^2\phi/\partial x_i \partial x_k$ is the gradient tensor of the electric field created by the surroundings at the center of the Mu, and the operator Q_{ik} represents the quadrupole moment of muonium:

$$Q_{ik} = C_F\left(\{F_i F_k\} - \frac{2}{3}\mathbf{F}^2\delta_{ik}\right)$$

$$C_F = \begin{cases} 0, & F = 0 \\ \frac{3}{2}\,eQ, & F = 1\,. \end{cases} \qquad (2)$$

Here i and k are coordinate indices (x, y, z), $\mathbf{F} = \mathbf{J} + \mathbf{S}$ is the total angular momentum of the muonium, and Q is its quadrupole moment in the crystal.

If the crystal is immersed in an external magnetic field B, the spin Hamiltonian of the Mu becomes

$$\hat{H} = A\mathbf{SJ} + \frac{1}{6} Q_{ik}\,\phi_{ik} + g_e\,\mu_e\,\mathbf{SB} + g_\mu\,\mu_\mu\,\mathbf{JB}\,, \qquad (3)$$

where g_e and μ_e (g_μ and μ_μ) are respectively the g-factor and magnetic moment of the electron (muon).

The behavior of the polarization of μ^+ mesons in Mu atoms is determined entirely by the structure of the energy levels corresponding to (3). These levels depend on the magnitude and orientation of **B** with respect to the principal axes of the gradient tensor of the electric field. Approximate expressions for the energy levels for an arbitrary orientation of these principal axes can be derived by perturbation theory under the assumption $A \gg \omega_e \gg |d|$. This approach leads to

$$E_0 = -\frac{3}{4}A - \frac{\omega_+^2}{A}, \quad E_1 = \frac{A}{4} + \frac{\omega_+^2}{A} - \frac{d}{4} f(\theta, \phi) \qquad (4)$$

$$E_{2,3} = \frac{A}{4} \pm \omega_- + \frac{d}{8} f(\theta, \phi)$$

where $\omega_\pm = (\omega_e \pm \omega_\mu)/2$, ω_e and ω_μ are the Larmor precession frequencies of the spins of the electron and the muon in the field **B**, $d = eQ\phi_{zz}$ is the quadrupole constant of Mu in a crystal, and the function $f(\theta,\phi)$ describes the quadrupole contribution to (3):

$$f(\theta, \phi) = 3\cos^2\theta - 1 + \eta\sin^2\theta\cos 2\phi, \qquad (5)$$

where θ and ϕ are the polar and azimuthal angles which specify the inclination of **B** with respect to the z axis of the gradient tensor of the electric field, and $\eta = |(\phi_{xx} - \phi_{yy})/\phi_{zz}|$.

The time dependence of the polarization vector of the μ^+ mesons is determined by the frequencies of all possible transitions between levels, $\omega_{ik} = E_i - E_k$. In a weak magnetic field, and if direct transitions between the singlet (E_0) and triplet (E_1, E_2, E_3) sublevels are not detected, the experimental μSR spectrum of the Mu precession will contain two frequencies,

$$\omega_1 = \omega_- - \frac{\omega_+^2}{A} + \frac{3}{8} df(\theta, \phi), \quad \omega_2 = \omega_- + \frac{\omega_+^2}{A} - \frac{3}{8} df(\theta, \phi). \tag{6}$$

Three groups have reported experiments[4–6] on the quadrupole interaction of Mu with matter. Until now, on the other hand, there have been no experiments in which the sign of the quadrupole constant d has been determined.

There is an interesting effect associated with the possibility of determining the sign of the constant d of muonium if the orientation of the principal axes of the gradient tensor of the electric field is known. Specifically, the difference between the frequencies in (6), $\Delta\omega = |\omega_1 - \omega_2|$, behaves in qualitatively different ways as functions of the magnetic field B for the different signs of d. Let us assume for definiteness that $f < 0$ (this case might correspond, for example, to a situation with $\theta = \pi/2$ and $\eta = 0$). Then with $d < 0$ the difference $\Delta\omega$ falls off quadratically with respect to B as this field is increased, and at a certain field value $B^* = ((3/4)dfA)^{1/2}/g_e|\mu_e|$ it vanishes and then begins to increase quadratically with the field (Fig. 1a). If $d > 0$, on the other hand, the difference $\Delta\omega$ can only increase monotonically with increasing B (Fig. 1b). It is thus sufficient to carry out several measurements near the field B^* in order to determine the sign of the quadrupole constant of muonium.

For the present measurements we used the Myuoniĭ apparatus[7] in a separated beam of positive muons in the muon channel of the synchrocyclotron of the Leningrad

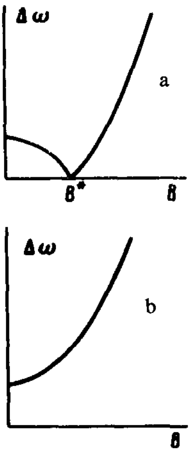

FIG. 1. Difference between the muonium precession frequencies, $\Delta\omega$, vs the external magnetic field **B** for opposite signs of the quadrupole constant d and a fixed orientation of the principal axes of the gradient tensor of the electric field. a—$d < 0$, $f < 0$; b—$d > 0$, $f < 0$.

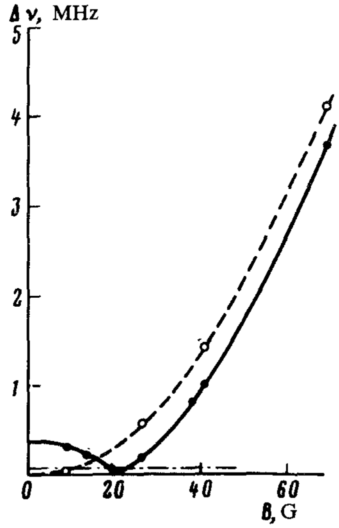

FIG. 2. Experimental results on the difference between the muonium precession frequencies $\Delta\nu$ (MHz) in crystalline quartz (filled circles) and in fused quartz (open circles) vs the external magnetic field **B** (in gauss). Dot-dashed line—experimental limit on the separation of the two precession frequencies; solid curve—a fit of the experimental points by expression (7) with the parameter values in (8). The parameter η is assumed to be zero.

Institute of Nuclear Physics[8] with a momentum ~ 108 MeV/c. The target consisted of three α-quartz single crystals. The $\hat{C}$ axis of the crystals coincided with the initial polarization direction of the muons and was perpendicular to the external magnetic field. Figure 2 shows the results of the measurements for a sample at room temperature. The filled circles show the measured frequency difference $\Delta\omega = |\omega_1 - \omega_2|$ as a function of the external magnetic field for α quartz. Shown for comparison by the open circles are measurements for fused quartz, taken under the same experimental conditions. The errors in the measurements of the frequency difference stem primarily from the statistical base and range from 0.01 to 0.02 MHz, in rough correspondence with the size of the circles in the figure. It can be seen from Fig. 2 that df is greater than zero; i.e., there are two versions: 1. $d>0, f>0$; 2. $d<0, f<0$.

To determine the sign of f we use the results of Ref. 4, where it was shown that at room temperature the hyperfine interaction has an axial symmetry and that the symmetry axis coincides with the $\hat{C}$ axis of the crystal. Assuming that the **z** axis of the gradient tensor of the electric field coincides with the $\hat{C}$ axis, we find $\theta = \pi/2$, $\eta = 0$, and $f = -1$. From (6) we then find

$$\Delta\omega = \left| \frac{2\omega_+^2}{A} + \frac{3}{4} d \right|, \quad d < 0 \tag{7}$$

The solid curve in Fig. 2 is a result of the fit of the experimental data on α quartz by expression (7). For the parameters A and d we find

$$A = 4510 \pm 30 \text{ MHz}$$

$$d = -0.517 \pm 0.008 \text{ MHz}. \tag{8}$$

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Translated by Dave Parsons
Edited by S. J. Amoretty