Possible ways to solve the triplet-doublet problem

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(Submitted 23 January 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 6, 280–282 (25 March 1984)

A supersymmetric SU(5) grand unified model, in which the triplet-doublet problem is solved in a natural way, is proposed. Generalizations of this model to the SO(10) and SU(n) gauge groups are discussed.

The triplet-doublet problem ranks along with the hierarchy problem as one of the most important and interesting problems in the grand unified theory.[1] Supersymmetric grand unified models[2] allow this problem to be solved "technically" (i.e., if the triplet-doublet problem is solved at the tree level, then radiation corrections will not alter the situation). The triplet-doublet problem can be summarized as follows: The masses of the quarks and leptons in the SU(5) grand unified model arise from a Yukawa interaction involving a 5-plet of Higgs fields. Under the SU(3) ⊗ SU(2) ⊗ U(1) subgroup of the SU(5) group, the 5-plet transforms as

$$5 = (3.1) + (1.2)$$

The (3.1) triplet is involved in processes which do not conserve baryon number; from the limits on the proton lifetime we find that the mass of the 3-plet satisfies[1] $M_3 \gtrsim 10^{11}$–10^{12} GeV. The (1.2) doublet, on the other hand, which is responsible for the electroweak symmetry breaking, must be essentially massless at the SU(5) unification scale. In the first stage of the symmetry breaking, SU(5)→SU(3) ⊗ SU(2) ⊗ U(1), which is conveniently arranged by means of nonzero vacuum expectation values of a 24-plet, we must have a heavy triplet (3.1) ($M_3 \gtrsim 10^{11}$–10^{12} GeV) and a massless doublet (1.2). It is far from a trivial matter to find such a huge splitting of the masses within a given 5-plet in some natural way. This is the triplet-doublet problem. It is solved in a more or less natural way in the triplet-mixing model,[3] which is one of the most popular models which have been advanced. In it, an additional 50-plet is introduced; as a result of the mixing of the triplets in the 5- and 50-plets, the triplets acquire large masses.

In this letter we wish to propose another mechanism for solving the triplet-doublet problem. We will discuss this mechanism in detail for the particular case of a supersymmetric SU(5) model, and we will outline the generalization to the cases of the SO(10) and SU(n) gauge groups. In addition to the standard 24-plet Ψ^{α}_{β} and the 5-plets H_{α} and $\overline{H}^{\beta}$, we introduce the singlet superfields σ and a. We introduce a reducible 25-plet $(\Psi^{\alpha}_{\beta}\sigma)\equiv\Phi^{\alpha}_{\beta})$. We choose the superpotential in the form

$$V(\Phi, H, \bar{H}, a) = h\bar{H}\Phi H + h_1 \operatorname{Tr}\Phi^3 + h_2 a \operatorname{Tr}\Phi^2 + M_3^2 a + h_4 a^3 . \tag{1}$$

The Lagrangian of this model is invariant under transformations

$$\Phi \to -\Phi, \quad a \to -a, \quad H \to -H. \tag{2}$$

The symmetry in (2) forbids a mass term $M\overline{H}H$ in superpotential (1). The equation for finding the minimum of the potential, $W = |(\partial V/2\phi_i)|^2$, has the nontrivial solution

$$\Phi = \Phi_0 \operatorname{Diag}(1,1,1,0,0),$$

$$\Phi_0^2 = \frac{-M_3^2}{3h_2 + 3h_4\left(\frac{3h_1}{2h_2}\right)^2} .$$

This solution corresponds to the symmetry breaking SU(5)→SU(3) ⊗ SU(2) ⊗ U(1). As a result of this symmetry breaking, the triplets $\overline{H}_3$ and H_3 acquire a mass $M_3 = h\Phi_0$, while the doublets $\overline{H}_2$ and H_2 remain massless at this stage of the breaking. The breaking of supersymmetry and of the electroweak SU(2) ⊗ U(1) symmetry in this model can be carried out in a standard way, by introducing terms which explicitly break the supersymmetry. The triplet-doublet problem is resolved in this model in a natural way [for all values of the parameters h_i and M_3^2 of superpotential (1), there exists a solution which leads to heavy triplets and massless doublets; in other words, there is no need to fine-tune the parameters].

We turn now to a generalization of this model to the case of the SO(10) gauge group. As super-Higgs fields we choose the 45-plet $\phi_{[ab]}$, the 10-plet H_a, and the 16-plets ξ_a and $\bar{\xi}^a$. The material fields are chosen in the standard way, as 3×16-plets M_a. Under the subgroup SU(5) ⊗ U(1) of the SO(10) group, the multiplets transform as

$$10 = 5(2) + 5(-2),$$

$$45 = 1(0) + 10(4) + 10(-4) + 24(0) .$$

The first stage in the symmetry breaking is arranged by means of nonvanishing vacuum expectation values of the 45-plet. We choose the nonvanishing vacuum expectation values in such away that

$$\langle 1(0) + 24(0) \rangle = \Phi_0 \operatorname{Diag}(1,1,1,0,0) . \tag{3}$$

For this vacuum expectation value we have $SO(10) \to SU(3) \otimes SU_L(2) \otimes SU_R(2) \otimes U(1)$. We consider the Lagrangian of the interaction of the 45-plet $\phi_{[ab]}$ and the 10-plet H_a:

$$\mathcal{L} = hH_a \phi_{[ab]} H_b \, . \tag{4}$$

A symmetry breaking of the type in (3) is accompanied by the appearance of a mass $M_3 = 2h\Phi_0$ for the triplets H_3 and $\overline{H}_3$, while the doublets H_2 and $\overline{H}_2$ remain massless. The mass term MH_aH_a can be forbidden by imposing a discrete symmetry $H_a \to iH_a$, $\phi_{[ab]} \to -\phi_{[ab]}$. Because of the nonvanishing vacuum expectation values $\langle \xi_a \rangle$ and $\langle \bar{\xi}^a \rangle$, the symmetry $SU(3) \otimes SU_L(2) \otimes SU_R(2) \otimes U(1)$ is broken to the standard $SU(3) \otimes SU(2) \otimes U(1)$ group, while the latter is broken by the nonvanishing vacuum expectation values $\langle H_2 \rangle$ and $\langle \overline{H}_2 \rangle$. To illustrate this mechanism for grand unified models based on SU(n) gauge groups, we consider the simple case of the SU(8) model. For superpotentials $V(\Phi) = V(-\Phi)$ [Φ^α_β is a 63-plet of the SU(8) group] there exists a solution $\langle \Phi \rangle = \Phi_0 \, \mathrm{Diag}\,(1, 1, 1, 0, 0, -1, -1, -1)$, which breaks SU(8) to $SU(3) \otimes SU(2) \otimes SU'(3) \otimes U(1) \otimes U'(1)$. Because of the existence of this solution, when the electroweak symmetry is broken by the 8-plets H_8 and $\overline{H}_8$ the presence of an interaction $h\overline{H}_8 \Phi H_8$ causes the triplets to acquire a mass $M_3 = h\Phi_0$, while the doublet remains massless.

I wish to thank V. A. Matveev and A. N. Tavkhelidze for useful discussions.

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Translated by Dave Parsons
Edited by S. J. Amoretty