Linear stage of the development of the instability of precession of magnetization in ^{3}He-*A*

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Equations are obtained for the change of experimentally observed quantities as a function of time at the initial stage of development of the instability of the spatially uniform precession of magnetization in ^{3}He-A.

The anomalously fast decay of the induction signal in pulsed NMR experiments with the A phase of ^{3}He accompanying the tilting of magnetization by large ($\simeq 90°$) angles away from the equilibrium orientation was observed experimentally and investigated by Borovik–Romanov *et al.*[1] The only currently possible explanation of this decay is the previously discussed[2] instability of the spatially uniform precession of magnetization in ^{3}He-A. The purpose of this paper is to determine the theoretically predicted time dependence of the experimentally observed quantities resulting from the development of the indicated instability.

As in Ref. 2, we shall assume that the magnetic field $\mathbf{H}_0$ is strong, i.e., the corresponding Larmor frequency ω_L is much larger than the frequency of longitudinal oscillations Ω. In this case, Leggett's equations,[3] averaged over times t such that $\omega_L^{-1} \ll t \ll \Omega^{-1}$,[4] can be used to describe the motion of the spin $\mathbf{S}$ and of the order parameter. The averaged motion of the system is then described by four variables—the spherical coordinates of the spin: $S = |\mathbf{S}|, \beta$, and α, as well as the angle ϕ, which determines the relative phase of precession and the rotation of the spin part of the order parameter around $\mathbf{S}$ (see Refs. 2 and 4). The averaged Leggett equations have a solution corresponding to spin precession with frequency $\omega_\perp(\cos\beta): S = S_0 = \text{const}$, $\beta = \beta_0 = \text{const}$, $\alpha = \alpha_0(t) = \omega_\perp t$, and $\Phi = \Phi_0 = 0$. This solution, as shown in Ref. 2, is unstable against violation of the spatial homogeneity, i.e., when the gradient terms are incorporated into the equations of motion, the perturbations which depend weakly on the coordinates $\phi(\mathbf{r})$, $\epsilon(\mathbf{r})$, $\nu(\mathbf{r})$, $\sigma(\mathbf{r})$, defined as $\Phi = \phi$, $\cos\beta = \cos\beta_0 + \epsilon$, $\alpha = \alpha_0 + \nu$, $S = S_0 + \sigma$, increase exponentially with time. Substitution of the Fourier expansion of the perturbations $\epsilon(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \epsilon_{\mathbf{k}}(t) e^{i\mathbf{kr}}$ etc. into the linearized equations of motion leads to the following values of the growth increment $\Gamma(k)$ and of the normal coordinates of the growing mode $\phi_{\mathbf{k}}, \epsilon_{\mathbf{k}}, \nu_{\mathbf{k}}$:

$$\Gamma(\mathbf{k}) = \left[\frac{C_{kk}}{4\omega_L^2} \left(\frac{3}{4}\Omega^2 \sin^2\beta - C_{kk} \right) \frac{\Omega^2 (3 - \cos\beta)(1 + \cos\beta) + 4C_{kk}}{\Omega^2 (1 + \cos\beta)^2 + 4C_{kk}} \right]^{1/2} - D_{kk}, \quad (1)$$

$$\phi_{\mathbf{k}} = \frac{1-\cos\beta}{2\sin^2\beta}\frac{C_{kk}}{\Omega^2}\frac{4C_{kk} - 3\Omega^2\sin^2\beta}{C_{kk} + \Omega^2(1+\cos\beta)^2} a_{\mathbf{k}}^{(0)} \exp[\Gamma(\mathbf{k})t],$$

$$\nu_{\mathbf{k}} = \left(\frac{C_{kk}}{2\Omega^2\sin^2\beta} - \frac{3}{8}\right) a_{\mathbf{k}}^{(0)} \exp[\Gamma(\mathbf{k})t], \tag{2}$$

$$\epsilon_{\mathbf{k}} = \frac{\omega_L \Gamma(\mathbf{k})}{\Omega^2} a_{\mathbf{k}}^{(0)} \exp[\Gamma(\mathbf{k})t].$$

The quantity σ_k is small for the significant values of $\mathbf{k}$. Here $C_{kk} = C^2_{\xi\eta}k_\xi k_\eta$, $D_{kk} = D_{\xi\eta}k_\xi k_\eta$, $C^2_{\xi\eta}$ is the tensor formed by the squares of the velocities of spin waves, $D_{\xi\eta}$ is the tensor formed by the coefficients of spin diffusion, and the general coefficient $a_{\mathbf{k}}^{(0)}$ gives the initial amplitudes of the perturbations with different values of $\mathbf{k}$.

The development of the instability leads to the loss of phase coherence of the spin precession; as a result, the volume-averaged value of the transverse component of the spin $\langle S_\perp\rangle$ decreases, consistent with the experimental results of Ref. 1. As long as the deviations from spatial uniformity are small, we can use them in an expansion

$$\langle S_\perp\rangle = S_\perp^{(0)}\left[1 - \frac{\cos\beta}{\sin^2\beta}\langle\epsilon\rangle - \frac{1}{2\sin^4\beta}\langle\epsilon^2\rangle - \frac{1}{2}\langle\nu^2\rangle\right]. \tag{3}$$

The resultant spatial inhomogeneity gives rise to superfluid and diffusion spin flows, which lead to a time-dependent average longitudinal spin component $\delta\langle S_\parallel\rangle = S\langle\epsilon\rangle$ and to an additional shift of the precession frequency $\langle d\nu/dt\rangle$. The equations describing the time dependence of these quantities at the initial stage of development of the instability can be obtained by averaging the equations of motion of the spin over the volume [see Eqs. (7)–(10) in Ref. 4]

$$\frac{d\langle\epsilon\rangle}{dt} = D_{\xi\eta}\left[\sin^2\beta\langle\nu_{,\xi}\nu_{,\eta}\rangle + \frac{1}{\sin^2\beta}\langle\epsilon_{,\xi}\epsilon_{,\eta}\rangle\right], \tag{4}$$

$$\left\langle\frac{d\nu}{dt}\right\rangle = -\frac{3}{8}\frac{\Omega^2}{\omega_L}\langle\epsilon\rangle + \frac{C^2_{\xi\eta}}{2\omega_L}\left[2\langle\nu_{,\xi}\phi_{,\eta}\rangle - (2-\cos\beta)\langle\nu_{,\xi}\nu_{,\eta}\rangle\right.$$

$$\left. + \frac{\cos\beta}{\sin^4\beta}\langle\epsilon_{,\xi}\epsilon_{,\eta}\rangle\right] - \frac{2\cos\beta}{\sin^2\beta}D_{\xi\eta}\langle\nu_{,\xi}\epsilon_{,\eta}\rangle. \tag{5}$$

Here $\epsilon_{,\xi} = \partial\epsilon/\partial x_\xi$, $\langle\epsilon^2\rangle = \frac{1}{V}\int[\epsilon(\mathbf{r})]^2 dV = \int[d^3k/(2\pi)^3]|\epsilon_{\mathbf{k}}|^2$ etc. To calculate the averages entering on the right sides of Eqs. (3)–(5) it is necessary to use Eqs. (2) and to take into account the fact that in the initial state the spatial nonuniformity is small and the perturbations reach an appreciable magnitude only for $\Gamma t \gg 1$, i.e., for long times.

The increment Γ has a maximum as a function of k, allowing the use of the saddle-point method to find the required averages. To simplify the calculations, we shall assume that the tensors $D_{\xi\eta}$ and $C^2_{\xi\eta}$ are proportional to each other, i.e., $D_{\xi\eta} = (\Lambda/2\omega_L)C^2_{\xi\eta}$. In this case, the maximum value $\Gamma = \Gamma_m$ is the same for all orientations of $\mathbf{k}$. It is convenient to find it numerically, making expression (1) dimensionless by substituting $C_{kk} = z(3/4)\Omega^2\sin^2\beta$. We then find

$$\Gamma_m = \Gamma(z_m) = \frac{3}{8}\,\frac{\Omega^2}{\omega_L}\sin^2\beta \left\{ \left[z(1-z)\,\frac{2+(3z+1)(1-\cos\beta)}{2+(3z-1)(1-\cos\beta)} \right]^{1/2} - \Lambda z \right\}\Bigg|_{z=z_m}, \tag{6}$$

where z_m is the value of z for which expression (6) reaches the maximum with respect to z. Substituting (2) into (3)–(5) and using the saddle-point method, we find the temporal dependences for the observed quantities

$$<S_\perp> = S_\perp^{(0)}\left[1 - \frac{A_S}{\sqrt{z_m^2\Gamma''(z_m)t}}\,e^{2\Gamma_m t}\right], \tag{7}$$

$$<\epsilon> = \frac{A_\epsilon}{\sqrt{z_m^2\,\Gamma''(z_m)\,t}}, \tag{8}$$

$$<\frac{d\nu}{dt}> = \frac{A_\delta}{\sqrt{z_m^2\Gamma''(z_m)\,t}}. \tag{9}$$

Thus the characteristic time of the buildup of perturbations is $\frac{1}{2}\Gamma_m$. This time depends on Ω^2/ω_L, Λ and $\cos\beta$. Detailed experimental data are available for Γ^2. The velocities of spin waves and the coefficients of spin diffusion have not been measured. Estimates based on theoretical values of C^2 and the value of D in the normal phase lead to $\Lambda \lesssim \frac{1}{2}$, for a pressure ~ 30 bar and $\omega_L = 2\pi\times 500$ kHz with $1 - T/T_c \gtrsim 0.1$ which gives $\frac{1}{2}\Gamma_m \approx 100$ μs. The pre-exponential factors $A_S, A_\epsilon, A_\delta$, which are found with the help of Eqs. (2), (3), and (5), depend on the amplitudes of the initial perturbations with wave vector $k_m \sim 10^2$ cm^{-1}. If it is assumed that thermal fluctuations existing before the tilting of the spin are the only source of such perturbations, then for A_S we obtain the value $\sim 10^{-6}$–10^{-7}.

The region of applicability of the equations obtained is limited by the condition that the corrections be small: $A\exp(2\Gamma_m t)/\sqrt{z_m^2\Gamma''(z_m)t} \ll 1$. The question as to the nature of the spatially inhomogeneous structure that appears in the nonlinear region when these corrections are not small has not yet been investigated theoretically. This is an important and interesting question.

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