Percolation regime in the reflection of electromagnetic waves from a metal–superconductor mixture

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Percolation processes in the skin layer cause the effective conductivity, which is part of the surface impedance of a metal–superconductor mixture, to have an anomalous frequency dependence and an anomalous temperature dependence.

The properties of an electromagnetic wave reflected from the plane surface of some material are known to be determined by the effective longitudinal conductivity σ. Since the wave is attenuated inside the medium, within a surface layer with a thickness on the order of the penetration depth λ, and since the screening current is also concentrated in this layer, the quantity σ is the effective conductivity of the medium over a length scale on the order of λ. For percolation media and also any fractal media, the electrical properties of the layer depend strongly on its thickness L (Ref. 1). In particular, the threshold for longitudinal percolation and thus the longitudinal conductivity depend on the thickness of this layer.

Let us consider a mixture of normal (metallic) and superconducting inclusions, with respective conductivities σ_n and σ_s satisfying the condition $|\sigma_n/\sigma_s| \ll 1$. We are interested in the properties of this medium in the case in which the concentration of superconducting inclusions is above the 3D percolation threshold, i.e., the case in which the dc resistance of the medium is zero. If the concentration of the superconducting phase is below the 2D percolation threshold, however, quasi-2D longitudinal percolation in a surface layer of thickness L will occur only under the condition $L > L_c$, where the critical thickness L_c depends on the concentrations of the phases.[1] Over length scales smaller than L_c we are dealing with an effectively homogeneous metal with isolated superconducting inclusions. Under the condition $L \gg L_c$, we are dealing with a "poor" superconductor with metallic inclusions. At $L \sim L_c$, the conductivity of the layer varies sharply with the layer thickness. Correspondingly, the effective longitudinal conductivity in our case is a function of the penetration depth: $\sigma = \sigma(\lambda)$. This penetration depth, on the other hand, is related to the conductivity by the standard formula

$$\lambda^{-2} = \mu_0 \omega |\sigma| \gamma^2, \quad \gamma = \sin\left(\frac{\pi}{4} + \frac{1}{2} \arg\sigma\right), \tag{1}$$

where $1/2 \leqslant \gamma^2 \leqslant 1$. Substituting the known expression for $\sigma(\lambda)$ from Ref. 1 into (1), we find a self-consistent equation for the effective conductivity, which is part of the surface impedance of this metal–superconductor mixture.

As a specific realization of this model we consider inhomogeneous high-T_c superconductors, many properties of which can be explained by a percolation model of the superconducting transition (Ref. 2, for example). For such high-T_c superconductors, both the relative amount of the superconducting phase and the conductivities σ_n and σ_s depend on the temperature. We assume that a two-fluid model with real σ_n is valid; we then have $\mathrm{Re}\,\sigma_s \leqslant \sigma_n$ and $\mathrm{Im}\,\sigma_s \sim \omega^{-1}$. The transition temperature T_s corresponds to 3D superconducting percolation in this case, while a quasi-2D percolation occurs at $T<T_s$. We consider frequencies for which the relations $a \ll \lambda < R \ll a|\sigma_s/\sigma_n|^{\varphi}$ hold. Here R is the 3D correlation radius of the percolation problem, a is the average size of an inhomogeneity, $\varphi = \nu/(s+t)$, and ν, s, and t are a correlation-radius index and conductivity indices for 3D space. A specific property of this model is that $\mathrm{Re}\,\sigma(\lambda)$ has a sharp maximum at $\lambda \simeq \lambda_c \equiv L_c$, while $|\sigma(\lambda)|$ varies monotonically. This conclusion follows directly from the asymptotic behavior of $\sigma(\lambda)$ near λ_c, which we will see below. Under the condition $\lambda < \lambda_c$ we have

$$\sigma \simeq \sigma_n(\lambda/a)^{s/\nu}[\tau(\lambda)]^{-z}, \quad \tau(\lambda) = \frac{p_{c2} - p(\lambda)}{p_{c2}}, \tag{2}$$

where z is the 2D analog of the index s, p_{c2} is the 2D percolation threshold, and $p(\lambda)$ is the effective concentration of the superconducting phase at the length scale[1] λ. This concentration increases with increasing λ. The asymptotic behavior in (2) is valid to

$$\tau(\lambda) > [(\lambda/a)^{1/\varphi}|\sigma_n/\sigma_s|]^F, \quad F = 1/(z+\theta), \tag{3}$$

where θ is a 2D analog of the index t. At smaller values of $\tau(\lambda)$, the conductivity is determined by the asymptotic behavior corresponding to $\lambda = \lambda_c$:

$$\sigma \simeq (\sigma_n, \sigma_s)^{1/2}(a/\lambda)^{\delta}, \quad \delta = \frac{t-s}{2\nu} \simeq 0.7. \tag{4}$$

The case $\lambda > \lambda_c$, in which $|\tau(\lambda)|$ also satisfies inequality (3), actually corresponds to[1] $\lambda \sim R$, so for such length scales we have

$$\sigma \simeq \sigma_s(a/R)^{t/\nu} + A\sigma_n(R/a)^{s/\nu}, \tag{5}$$

where A is a positive constant on the order of unity.

From the experimental standpoint, the percolation regime of the reflection of electromagnetic waves from a disordered superconductor is seen most clearly in the temperature dependence and the frequency dependence of the effective longitudinal conductivity. The real part $\mathrm{Re}\,\sigma(T)$ has a clearly defined maximum corresponding to $\lambda = \lambda_c$, and the conductivity itself satisfies the following equation at this point [see (1) and (4)]:

$$\sigma = \exp\left(\frac{i\pi}{4}\right)[(\gamma^2 a^2 \mu_0 \omega)^{\delta}|\sigma_n \sigma_s|]^{1/(2-\delta)}, \quad \gamma = \sin(3\pi/8). \tag{6}$$

The conductivity has a nontrivial frequency dependence, which is governed by the index δ. Since λ_c depends on the relative concentration of the phase and therefore on the temperature, the position of the maximum of $\mathrm{Re}\,\sigma$ is determined by the equation

$$\lambda_c(T_{\max}) = [a^{\delta}\gamma^2 \mu_0 \omega |\sigma_n \sigma_s|^{1/2}]^{-1/(2-\delta)}. \tag{7}$$

The characteristic half-width ΔT of the maximum is determined by the condition that $\lambda_c(T_{\max} - \Delta T)$ be equal to the depth to which the electromagnetic wave penetrates into the sample at $T \simeq 0 - \lambda(0)$:

$$-[p_{c3}-p(T_{\max}-\Delta T)]=(p_{c2}-p_{c3})[\lambda(0)/a]^{1/\nu}. \qquad (8)$$

Here $p(T)$ is the relative concentration of the superconducting phase at the given temperature, and p_{c3} is the 3D percolation threshold.

A peak in $\mathrm{Re}\,\sigma$ similar to that discussed above was observed by Klein *et al.*[3] in measurements of the surface impedance of a YBaCuO crystal. Klein *et al.*[3] attempted to interpret this peak as a coherence peak, using an adjustable parameter, but it was shown in a recent review[4] that the true nature of conductivity anomalies of this type in high-T_c superconductors has not been resolved. In the case of thin films, for example, the existence of a peak in $\mathrm{Re}\,\sigma$ would follow directly from the 2D percolation model of the superconducting transition.

At a qualitative level, the results of Ref. 3 agree with the percolation-reflection regime discussed above. In particular, the sample thickness was much larger than the depth of penetration into any of the phases, the transition temperature in terms of the resistance was 92 K, and the peak was observed at $T_{\max} \simeq 89$ K. Since we do not have information on the function $p(T)$ for that experiment, we can offer only a crude estimate of the width of the peak, based on data on the temperature dependence of the surface impedance $Z_s = R_s + iX_s$. At 92 K we have $R_s \simeq X_s$, and the test sample is in the normal phase. At 86 K we have $R_s/X_s \simeq 0$; i.e., the sample is an essentially 100% superconductor. It was in this interval that the peak in $\mathrm{Re}\,\sigma$ was observed. For X_s/R_s at $T = T_{\max}$, expression (6) yields a value of 2.4; the experimental value is 2.8. From the temperature dependence of Z_s we also have $X_s(95\ \mathrm{K})/X_s(86\ \mathrm{K}) \simeq 5$; using the two-fluid model, we then find $\mathrm{Re}\,\sigma(T_{\max})/\sigma_n \simeq 4(a/\lambda)^{\delta}$, where σ_n is the conductivity of the normal phase at 95 K. From (6) we have $\mathrm{Im}\,\sigma = \mathrm{Re}\,\sigma$ at $T = T_{\max}$; we can thus determine λ from (1). Using the experimental value of $\mathrm{Re}\,\sigma(T_{\max})/\sigma_n \simeq 2.2$, we can thus estimate the average size of the inhomogeneity; we find $a \sim 0.3\ \mu$m. This result agrees with, for example, an estimate of a corresponding quantity in Ref. 5. However, the most reliable way to compare the predictions of the present study with experimental data would be to measure the frequency dependence of the magnitude and position of the maximum of $\mathrm{Re}\,\sigma$. These properties are described by expressions (6) and (7).

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