Effect of rotation on the NMR signal in the helium-3 *A* phase

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The broadening of the NMR line in the *A* phase of helium-3, which results from the excitation of spin waves at vortices formed as the vessel is rotated, is derived for the case of a random arrangement of vortices. The possibility of observing changes in the NMR spectrum associated with an ordering of the system of vortices is discussed.

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Rotating a vessel holding superfluid helium-3 should, as in the case of helium-4, result in the formation of a system of vortices which stimulates, on the average, a rigid-body rotation of the helium.[1] In a recent experiment, Hakonen *et al.*[2] used a continuous NMR method to detect a change caused in the state of the helium-3 *A* phase by rotation. In particular, they observed a change in the absorption level at the line maximum and studied the dependence of this change on the temperature and the velocity at which the vessel was rotated. An unambiguous interpretation of these results requires a theoretical analysis of the changes caused in the NMR spectrum by the presence of vortices. Volovik and Hakonen[3] have shown that vortices should give rise to a satellite on the low-frequency side of the fundamental line. This satellite does not, however, exhaust the list of possible changes in the NMR spectrum. Vortices render the liquid spatially inhomogeneous; as a result, a spatially uniform rf field can excite spin waves, which would be seen as a broadening of the NMR line and a reduction in its height. In this letter we will derive a theory for this problem.

Under the conditions corresponding to continuous NMR, the motion of the magnetization in superfluid helium-3 is described by the linearized Leggett equations[4] with a driving force on the right side. The energy absorption measured in the laboratory is determined by the imaginary part of the Green's function of this system of equations. We will be discussing a transverse resonance everywhere below. After a transformation to normal coordinates, the Green's function for the transverse motion of the magnetization in the absence of vortices is written in its usual form:

$$G^{(0)}(\omega, k) = [\omega - \omega_{\perp}(k) + i\Gamma_{\perp} + iDk^2]^{-1}.$$

For strong magnetic fields, for which the Larmor frequency ω_L is large in comparison with the longitudinal oscillation frequency Ω, we have $\omega_{\perp}(k) = \omega_L + (\Omega^2 + c^2k^2)/2\omega_L$. For simplicity, we are ignoring the tensor nature of the spin-wave velocity c and of the spin-diffusion coefficient D; here $\Gamma_{\perp}$ is the width of the transverse-resonance line at $k = 0$. Each vortex is a coordinate-dependent potential $w(\mathbf{r})$[3] for the moving magnetization; the scale value of this potential is $\sim\Omega^2/\omega_L$, and its range is $\sim l_{\Omega} \sim c/\Omega \sim 10^{-3}$ cm. At the rotation velocities currently attainable, ~ 1 rad/s, the average distance between the vortices is at least an order of magnitude greater than l_{Ω}. The equilibrium configuration of the system of vortices should be a regular array, but a long time may be required to reach equilibrium, so that we should assume that the arrangement of vortices under the experimental conditions of Ref. 2 was approximately random. If we assume that the arrangement is completely random, then we can find the change in the Green's function due to the vortices by the familiar technique of averaging over impurities.[5] The leading terms in the perturbation-theory series are represented by the diagrams in Fig. 1. The Green's function found by summing this sequence of diagrams is

$$G(\omega, k) = [\omega - \omega_{\perp}(k) + i\Gamma_{\perp} + iDk^2 + \Sigma(\omega, \mathbf{k})]^{-1}, \tag{1}$$

for which the Born-approximation expression for the energy part proper is

$$\Sigma(\omega, \mathbf{k}) = \Sigma_1 + i\Sigma_2 = \frac{n}{(2\pi)^2}\int |w(\mathbf{k} - \mathbf{k}')|^2 G(\omega, \mathbf{k}')\, d^2k'. \tag{2}$$

Here n is the number of vortices per cm^2, and $w(\mathbf{k})$ is the Fourier transform of the potential $w(\mathbf{r})$. Since the wavelengths of importance to this problem are on the order of the distance between the vortices, i.e., much larger than the range of the potential, we may set $w(\mathbf{k} - \mathbf{k}') = w_0 = \text{const} \sim \Omega^2 l_{\Omega}^2/\omega_L$. In this case the integral on the right side of (2) diverges and should be cut off at wavelengths $\sim l_{\Omega}$. The argument of the logarithm contains the large quantity $1/(nl_{\Omega}^2)$. The sequence of diagrams shown in Fig. 1 is the leading sequence in the logarithmic approximation in this parameter. Since the motion of the magnetization is excited by a uniform field of frequency ω, the absorbed power is $I \sim \mathrm{Im}\, G(\omega, k = 0)$, and the absorption at the line maximum is $I_{\max} \sim [\Gamma + \Sigma_2(\omega, 0)]^{-1}$. Here Γ is the total line width in the absence of vortices; it consists of $\Gamma_{\perp}$ and Γ_H, which is the width caused by the nonuniformity of the magnetic field in the volume

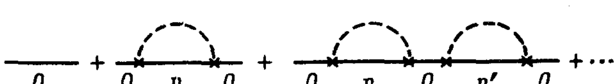

FIG. 1.

under study. For the additional width due to the vortices (Σ_2) we find from (1) and (2)

$$\Sigma_2 = -\frac{n\,|w_0|^2}{4\pi[D^2 + (c^2/2\omega_L)^2]}\left[D\mathcal{L} + \frac{c^2}{2\omega_L}\varphi\right], \tag{3}$$

where $\mathcal{L}$ and φ are respectively the real and imaginary parts of $\ln[(c^2/2\omega_L - iD)k^2_{\max}/(\omega - \Sigma_1) + i(\Gamma_\perp - \Sigma_2)]$, and $\mathcal{L}$ can be assumed equal to 4-5 over the entire range of parameters. We will not need an expression for Σ_1 here. To transform from the Born approximation to the general case, we should replace w_0 in (3) by the total scattering amplitude. This total amplitude is not known, so we will use an estimate for the Born amplitude.

We thus see that Σ_2 is proportional to the number of vortices, i.e., to the angular velocity of the rotation. To clarify the temperature dependence of Σ_2, we note that c^2 is proportional to τ, where $\tau = 1 - T/T_c$, and $|w_0|^2 \sim \tau^2$. According to data on the normal phase,[6] the value of D at $T = T_c$ is 0.02 cm^2/s. In the superfluid phase, near T_c, we should assume $D = D(T_c) - D_1\sqrt{\tau}$; the coefficient of $\sqrt{\tau}$ is not small, and D may vary by a factor of several units in a narrow region near $\tau \sim 10^{-2}$. If we assume $c \approx 5$ m/s (Ref. 7) near the transition to the B phase, then the absorption is determined by diffusion at $\tau \lesssim 0.04$. In this region the temperature dependence of Σ_2 should be of the type $\tau^2/[D(T_c) - D_1\sqrt{\tau}]$. At $\tau \approx 0.04$ we have $c^2/2D\omega_L \approx 1$, but the condition $2\mathcal{L}D\omega_L \gg c^2$ still holds, and in the region $2\mathcal{L}D\omega_L \gg c^2 \gg 2D\omega_L$ the value of Σ_2 is essentially independent of the temperature. Even further from T_c ($\tau \gtrsim 0.15$), we have $c^2 \gtrsim 2\mathcal{L}D\omega_L$; the diffusion becomes inconsequential, and we have $\Sigma_2 \sim \tau$.

The observed temperature dependence of Σ_2 exhibits these characteristic regions.[2] Also, the calculations yield values for Σ_2 in approximate agreement with the measured values, so that we may say that this theory agrees qualitatively with experiment. Lacking more accurate data on the magnitude and temperature dependence of D and c^2, and also w_0, we cannot make a quantitative comparison at this time. A more thorough experimental study of the temperature dependence of Σ_2 would make it possible to refine these estimates and to find more evidence for the assertion that the observed broadening of the NMR line is caused by the vortices.

Another important circumstance is that there exists a region with $c^2 \gg 2D\omega_L$. In principle, it would be possible to observe an ordered array of vortices in this region. In this case the NMR spectrum should consist of a set of lines with a spacing $\Delta\omega \sim (\pi^2 c^2 n/2\omega_L$ which corresponds to the Bragg scattering of spin waves by the vortex array. For rotation velocities ~ 1 rad/s, the corresponding splitting would be $\Delta\omega \sim 10^3\ \text{s}^{-1}$. This splitting would be resolvable in the most uniform magnetic field used in the experiments of Ref. 2. A study of this spectrum would make it possible to carry out a structural analysis of the vortex array and to measure the spin-wave velocity in the helium-3 A phase.

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