Microscopic quantum tunneling in Josephson junctions

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The effect of dissipation on microscopic quantum tunneling in Josephson junctions is studied. It is shown that dissipation increases the lifetime of the metastable state of the junction. It is found that the tunneling probability can be clearly separated into quantum-mechanical tunneling in the absence of dissipation and dissipation-related tunneling only for small dissipation.

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In the absence of fluctuations, the average constant voltage on a Josephson junction $\langle V \rangle = 0$, if the current I flowing through the junction is less than the critical current I_c. Fluctuations cause transitions of the Josephson phase shift φ between neighboring minima in the potential energy of the junction $U(\varphi)$ and lead to the appearance of the voltage $\langle V \rangle \neq 0$ even in the case $I < I_c$. For high temperatures exceeding the characteristic frequency of oscillations near the metastable minimum ω_0 and for any dissipation γ, the probability for overcoming the barrier is proportional to $\exp(-\Delta U/T)$, (ΔU is the barrier height). Quantum mechanical tunneling through the barrier is possible for $T \to 0$ ($T \ll \omega_0$) and a low height of the barriers separating the neighboring minima in $U(\varphi)$, i.e., when the current I is close to I_c ($\epsilon = 1 - I/I_c \ll 1$). The appearance of such tunneling has apparently been observed in recent experiments[1–4] on small Josephson junctions with a high current density. This effect was predicted theoretically ignoring dissipation by Ivanchenko and Zil'berman[5] (see also the later papers[4,6]). A theory, which includes dissipation with tunneling, was developed in Ref. 7. In Ref. 7, the limit $T = 0$ was examined and the calculation was performed for weak dissipation. In addition, in a recently published paper,[8] it was concluded on the basis of a model of tunneling through a parabolic barrier that the results in Ref. 7 are incorrect. For this reason, it is interesting to investigate the problem of microscopic quantum tunneling in greater detail. Here we shall examine the case in which dissipation is not too small $\gamma \gtrsim \omega_0$ and we shall show that in this case a simple separation of tunneling into a purely quantum-mechanical part with $\gamma = 0$ and a part arising from dissipation is incorrect.

Let us introduce the required notation. For Josephson's system $\gamma = (RC)^{-1}$, $\omega_0 = \omega_j (2\epsilon)^{1/4}$, R and C are the resistance and capacitance of the junction, $\omega_j^2 = 2\pi I_c / (\Phi_0 C)$, and $\Phi_0 = 2.07 \times 10^{-15}$ W is the flux quantum. Perturbation theory is valid for $\gamma \ll \omega_0$. The resistance R is a function of temperature[9]:

$$R = R_N \operatorname{ch} \frac{\Delta}{2T} \left[1 + \frac{\Delta}{8T} \ln \left(\frac{T}{V} \right)^2 \right]^{-1},$$

where R_N is the resistance of the junction in the normal state. V is a low voltage across the junction, and Δ is the energy gap between the superconducting boundaries. R_N

increases on exponentially if $T \to 0$ ($\gamma \to 0$). For low-resistance junctions, the value of γ may still be large in the temperature range where quantum-mechanical tunneling can occur.

Tunneling probability is calculated by methods developed in Refs. 7, 10, and 11. We represent the vacuum-vacuum transition amplitude inthe Euclidean formulation in terms of the Feynman path integral and we shall calculate it in the quasiclassical approximation. We shall link the friction coefficient γ to the linear interaction of the Josephson oscillator with the medium (thermostat). After averaging over the thermostat variables, the effective action assumes the form

$$S_{\text{eff}}\ (x(\tau)) = \int_{-\beta/2}^{\beta/2} d\tau \left[\frac{M}{2}\left(\frac{dx}{d\tau}\right)^2 + \frac{M\omega_j^2\sqrt{2\epsilon}}{2} x^2(\tau) - \frac{M\omega_j^2}{6} x^3(\tau)\right] + S_1(x(\tau)), \tag{1}$$

where

$$S_1(x(\tau)) = \frac{1}{2} \int\int_0^{\beta} \tilde{\alpha}(\tau - \tau')(x(\tau) - x(\tau'))^2 d\tau d\tau',$$

$$\tilde{\alpha}(\tau) = \frac{1}{2\pi}\int_0^{\infty} d\omega\, J(\omega) \frac{\text{ch}\ \frac{\omega\beta}{2}(1 - 2T|\tau|)}{\text{sh}(\omega\beta/2)}, \quad M = C(\Phi/2\pi)^2, \quad \beta = 1/T.$$

$J(\omega)$ is the spectral density: $J(\omega) = \gamma M\omega$ for $\omega < \omega_c$, where ω_c is the limiting frequency ($\omega_c \gg \omega_j$). In the low-temperature limit, $T \to 0$: $\tilde{\alpha}(\tau) = M\gamma/(2\pi\tau^2)$, ($\tau\omega_c \gg 1$), $\alpha(0) = \gamma M\omega_c^2/(4\pi)$.

In the quasi-classical approximation, the tunneling probability is proportional to $\exp[-(1/\hbar)S_{\text{eff}}(x_c(\tau))]$. The trajectory $x_c(\tau)$ satisfies an equation following from the stationary action (1) and boundary conditions $x_c(\tau) \to 0$ for $\tau = \pm\beta \to \infty$ and $\partial x(0)/\partial\tau = 0$.[10,11] The exact solution $x_c(\tau)$ of this equation cannot be found in general. However, it is possible to use the variational method with a trial function of the form

$$x_c(\tau) = \sqrt{2\epsilon}\, a\, \text{ch}^{-2}(\omega_j\rho\tau/2) \tag{2}$$

with two parameters a and ρ, which are determined from the condition that the action remain stationary. The quantity a determines the turning point $x_c(0) = a\sqrt{2\epsilon}$ on the right in Fig. 1. For finite β, the left turning point will be farther to the right than indicated and for $\beta \to \infty$ it will approach $x = 0$. In he case $S_1 = 0, a = 3, \rho = (2\epsilon)^{1/4}$, Fig. 1 shows the dependence of the potential energy $\tilde{U}(x)$ [sum of the second and third

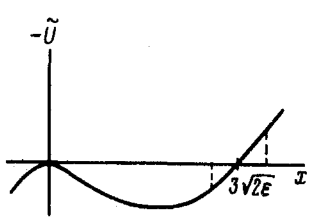

FIG. 1. The potential energy (with a minus sign) as a function of the coordinate x.

terms in square brackets on the right side of Eq. (1) taken with a minus sign] normalized to the quantity $M\omega_0^2$ as a function of $x(\tau)$.

Solving the system of equations satisfied by the variational parameters a and ρ, we obtain

$$a = 3(1-\alpha^2/2) \pm \frac{3}{2}\alpha\sqrt{1-\alpha^2}\,; \quad \rho = 5\sqrt{2\epsilon}\left(1-\frac{4}{15}a\right). \tag{3}$$

Here we introduce the following notation: $\alpha = 3\gamma\widetilde{A}_0/(4\omega_0), \omega_0 = \omega_j(2\epsilon)^{1/4}$

$$\widetilde{A}_0 = \frac{1}{4\pi}\int\int_{-\infty}^{\infty} \frac{dtdt'}{(t-t')^2}\left(\frac{1}{\mathrm{ch}^2 t} - \frac{1}{\mathrm{ch}^2 t'}\right)^2 f\left(\frac{\omega_j\rho}{2|t-t'|}\right) \tag{4}$$

$$f(z) = \frac{1}{z\gamma M}\int_0^{\infty} dx J(x,z)e^{-x} \simeq 1.$$

i.e., $\widetilde{A}_0 = A_0$, where A_0 is the constant introduced in Ref. 7, $A_0 = (12/\pi^3)\zeta(3)$. In order to choose the correct value of a from (3), it is necessary to examine the change in energy as the system moves away from the right-hand turning point. The energy $E(\tau)$ is determined from the equation $E = (M/2)(dx/d\tau)^2 - U(x)$. The total change in the energy ($\tau \to \infty$) over the time of motion from the right-hand turning point to the top of the left-hand maximum ($\tau \to \infty$) (see Fig. 1) can easily be calculated. In this case, $\Delta E < 0$ and the particle energy ρ decreases when the particle reaches the left-hand maximum in the potential energy. In order to compensate for this loss, it is necessary to choose a more distant turning point, which corresponds to the solution (3) with a plus sign. The final expression for S_{eff} assumes the form (see Fig. 2):

$$S_{\text{eff}} = \frac{18\text{v}_0}{\omega_0}\left(1+\frac{\alpha}{2}\left(\sqrt{1+\alpha^2}-\alpha\right)\right)^2\left(\frac{2}{5}\sqrt{1-2\alpha\sqrt{1+\alpha^2}-\alpha)}+\alpha\right), \tag{5}$$

where we introduced the barrier height $\text{v}_0 = (2/3)M\omega_j^2(2\epsilon)^{3/2}$ and the frequency associated with damping $\omega_\gamma = \omega_0^2/\gamma = 2eI_cR\,(2\epsilon)^{1/2}/\hbar, (\alpha = (3/4)\omega_0 A_0/\omega_\gamma)$. Thus, if friction is not small, then there is no clear separation of tunneling into quantum-mechanical and dissipative tunneling, and the tunneling probability decreases. This does not agree with the assertions made in Ref. 8. The proof in Ref. 8 is based on tunneling through a parabolic barrier. In this case, the tunneling probability is $p \sim e^{-\text{v}_0\tau_0}$, where $\tau_0 = 2\pi/\omega_j$. Here dissipation indeed increases $\omega_j\,(\omega_j \to \widetilde{\omega}_j > \omega_j)$ and for this reason, the

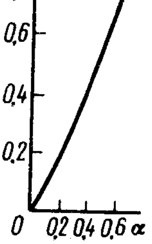

FIG. 2. $\widetilde{S}$ as a function of α (where $\widetilde{S} = (S_{\text{eff}}(\alpha) - S_{\text{eff}}(\alpha=0))$: $(18\text{v}_0/\omega_0)$.

quantity p as well. This is not the case, however, for the nonlinear problem with a term x^3 in the action. In reality, the current, which is fixed for the Josephson junction in the medium from the very beginning, is renormalized. For this reason, renormalization effects correspond to taking into account the real current in a Josephson junction.

If $\gamma \gg \omega_0$, then a different trial function is more appropriate then (2). It follows from the equation for a stationary trajectory that

$$\tilde{x}_c(\tau)_{\tau\to\infty} = \frac{2\tilde{\alpha}(\tau)}{M\omega_0^2}\int_{-\beta/2}^{\beta/2} \tilde{x}_c(\tau')d\tau' \equiv \frac{1}{\tau^2}A, \quad x_c(\tau) = \tilde{c}\,\tilde{x}_c(\tau).$$

We choose a trial function of the form $x_c(\tau) = \tilde{c}A/(\tau^2 + A)$ and determine $\tilde{c}$ from the condition that the action is stationary. We obtain for $S_{\rm eff}$

$$S_{\rm eff} = 4M\epsilon\,\pi\left(\omega_\gamma + \frac{\omega_0^2}{\omega_\gamma}\right) = \frac{3\pi v_0}{\omega_0}\left(\frac{\omega_\gamma}{\omega_0} + \frac{\omega_0}{\omega_\gamma}\right). \tag{6}$$

Thus, the main contribution to tunneling is related to dissipation. We note that if in Eq. (5) we set $\alpha \gg 1$, then it assumes the form $\simeq 27\alpha v_0/\omega_0 \simeq 3\pi v_0/\omega_\gamma$ and is close to the value following from Eq. (6). The presence of dissipation also leads to renormalization of the characteristic oscillation frequency $\omega_0 : \omega_c \to \tilde{\omega}_0 = \omega_j\,\rho = \omega_0[1 - 2\alpha(\sqrt{1+\alpha^2} - \alpha)]^{1/2}$, i.e., $\tilde{\omega}_0$ decreases. This corresponds to the fact that the temperature at which quantum-mechanical tunneling is important also decreases $(T < \tilde{\omega})$.

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[1]R. De Bruyn Ouboter and D. Bol, Physica **107** (B + C), 965 (1981).
[2]L. D. Jacket, J. P. Gordon, F. L. Hu, R. E. Howard, L. A. Fetter, D. M. Tennant, R. W. Epworth, and J. Kurkijarvi, Phys. Rev. Lett. **47**, 697 (1981).
[3]R. F. Voss and R. A. Webb, Phys. Rev. Lett. **47**, 265 (1981).
[4]R. H. Koch, D. J. van Harlingen, and J. Clarce, Phys. Rev. Lett. **45**, 2132 (1980).
[5]Yu. M. Ivanchenko and L. A. Zil'berman, Zh. Eksp. Teor. Fiz. **55**, 2395 (1968) [Sov. Phys. JETP **28**, 1272 (1969)].
[6]K. K. Likharev, Physica **107** (B + C), 1079 (1981).
[7]A. O. Caldeira and A. J. Legget. Phys. Rev. Lett. **46**, 211 (1981).
[8]A. Widom, and T. D. Clark, Phys. Rev. Lett. **48**, 63 (1982).
[9]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **53**, 2159 (1967) [Sov. Phys. JETP **26**, 1219 (1968)].
[10]S. Coleman, Phys. Rev. D **15**, 2929 (1977).
[11]S. G. Callan Jr. and S. Coleman, Phys. Rev. D **16**, 1762 (1977).

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