Influence of vacuum polarization by strong magnetic field on the cyclotron radiation of a thermal plasma

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It is shown that for the condition (1) the polarization of an electron-positron vacuum by a magnetic field alters qualitatively the spectrum, polarization and angular distribution of the cyclotron radiation of a plasma.

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1. It has been shown[1-3] that for the condition:

$$V = \frac{1}{60\pi N}\left(\frac{mc}{\hbar}\right)^3\left(\frac{\hbar\omega_B}{mc^2}\right)^4 = \left(\frac{4.5\times10^{28}\mathrm{cm}^{-3}}{N}\right)\left(\frac{B}{B_c}\right)\gtrsim 1\,, \tag{1}$$

where N is the electron density, B is the magnetic field, $\omega_B = eB/mc$, $B_c = 4.4\times10^{13}$ G, the virtual production processes of electron-positron pairs (vacuum polarization) exerts a considerable influence on the emission and propagation of electromagnetic waves in a cold ($T \ll mc^2$) plasma. As a result anomalies arise in the spectrum (such as absorption and emission lines), the polarization and the angular distribution of the plasma radiation. Vacuum polarization effects can be observed in the x-ray radiation of neutron stars ($B \sim 10^{11}-10^{13}$ G). In the laboratory similar effects, associated with the polarization of the electron-hole "vacuum", can appear in semiconductors,[4] where the critical field $B_c \sim 10^7$ G is determined by the forbidden band gap rather than the threshold for electron-positron production.

The results of Refs. 1–3 are not applicable in the cyclotron resonance region $|\omega - l\omega_B| \lesssim \omega\beta|\cos\theta|$, $\beta = (2T/mc^2)^{1/2}$. For this region expressions are obtained in this paper for the absorption coefficients, the indexes of refraction, and the polarizations of normal waves (NW) for $\hbar\omega_B < T$. It follows from these that when condition (1) exists, the vacuum polarization qualitatively alters the spectrum, polarization and angular distribution of the cyclotron radiation of a collisionless plasma. These changes must be taken into account, for example, in the interpretation of the spectral anomalies,[5] identified with cyclotron lines, in the radiation of a number of x-ray pulsars.

2. In a coordinate system with the Z axis along the wave vector and with the vector **B** in the ZOY plane the vacuum polarizability[6] for $(\hbar\omega/mc^2)(B/B_c) \ll 1$ is:

$$a_{xx}^{(V)} = a\sin^2\theta,\quad a_{yy}^{(V)} = \frac{7}{4}a\sin^2\theta,\quad a_{xy}^{(V)} = a_{yx}^{(V)} = 0, \tag{2}$$

where $a = e^2/45\pi^2\hbar c)(B/B_c)^2$, θ is the angle between the radiation propagation and magnetic field directions. Combining (2) with the polarizability of a thermal plasma[7] and assuming the plasma to be rarefied $[\omega_p^2 = 4\pi Ne^2/m \ll \omega^2\beta(|\cos\theta| + \beta)]$, we ob-

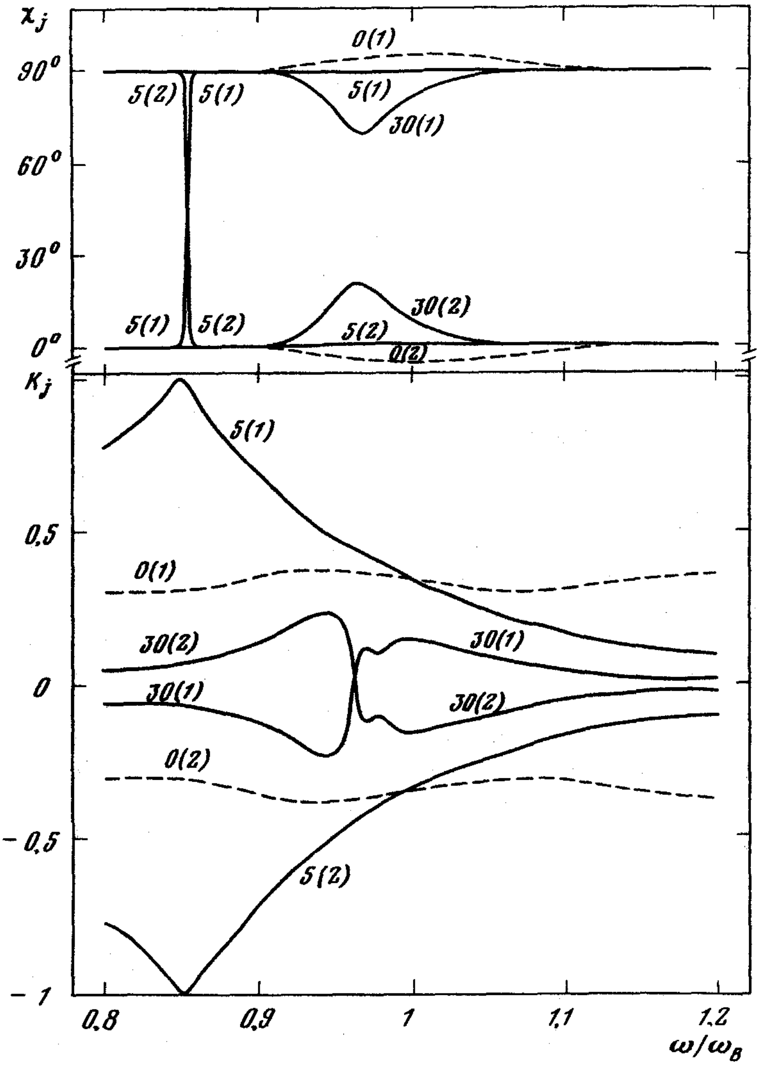

FIG. 1. Ellipticity K_j and positional angles χ_j of NW polarization ellipses in the vicinity of the first cyclotron resonance at $\theta = 70°$. Here and in the rest of the figures $\beta = 0.14$; the numbers near the curves denote the value of V and the number of the NW $j = 1, 2$ (in parentheses).

tain in the usual manner the index of refraction κ_j and the absorption coefficient $(2\omega/c)k_j$ of normal waves for the "vacuum + plasma" system:

$$n_j = \kappa_j + ik_j = n_I \pm (n_L^2 + n_C^2)^{1/2},$$

$$n_I = 1 + \pi(a_{xx} + a_{yy}), \quad n_L = \pi(a_{xx} - a_{yy}); \quad n_C = 2\pi i\, a_{xy} \tag{3}$$

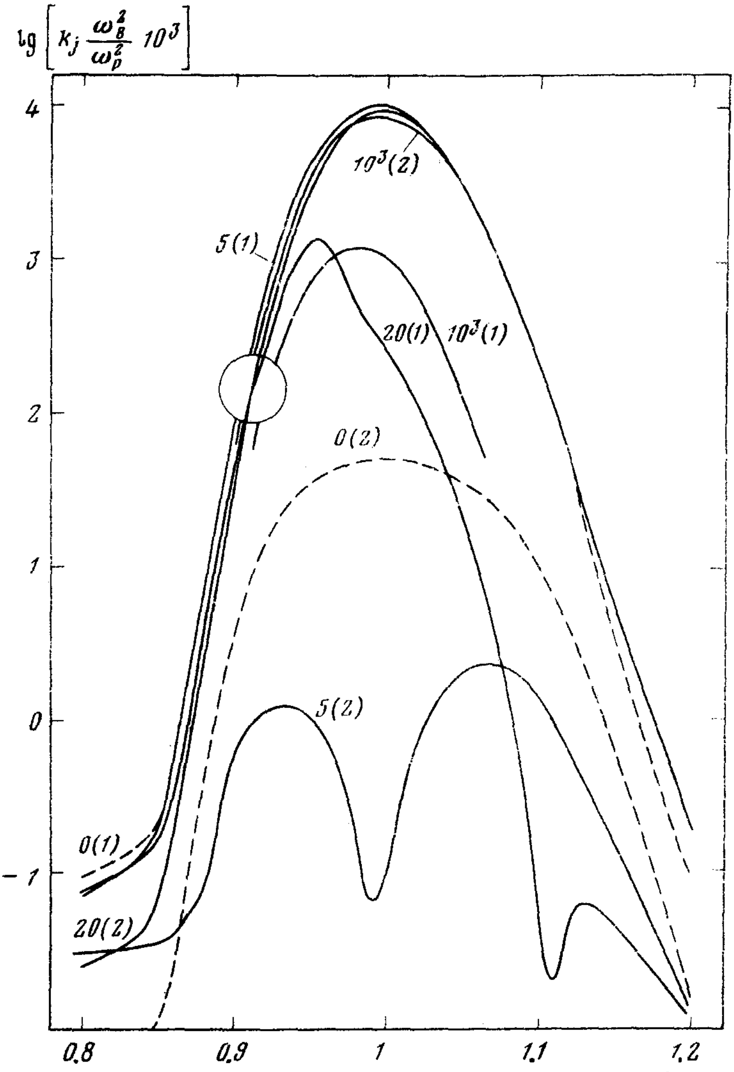

FIG. 2. The absorption coefficients k_j of the NW in the vicinity of the first resonance at $\theta = 70°$. Circle indicates intersection region of k_j.

The polarization of the jth NW is characterized by the ratio $|K_j|$ of the minor axis of the polarization ellipse to the major (the ellipticity), by the positional angle χ_j between the major axis of the ellipse and the Y axis and by the rotation direction of the electric vector (sign K_j), which are expressed in terms of n_L, n_C.[3]

3. An analysis of Eqs. (1)–(3) makes it possible to draw the following conclusions.

a) Vacuum polarization qualitatively alters the NW polarization in the vicinity of the first cyclotron resonance $\omega = \omega_B$ (Fig. 1). If $\chi_j \approx 0$, $\pm 90°$ in a vacuum-free plasma, and the K_j change little in the region $|\omega - \omega_B| \lesssim |\cos\theta|$, then for $V = 3\pi a \omega_B^2/\omega_p^2$

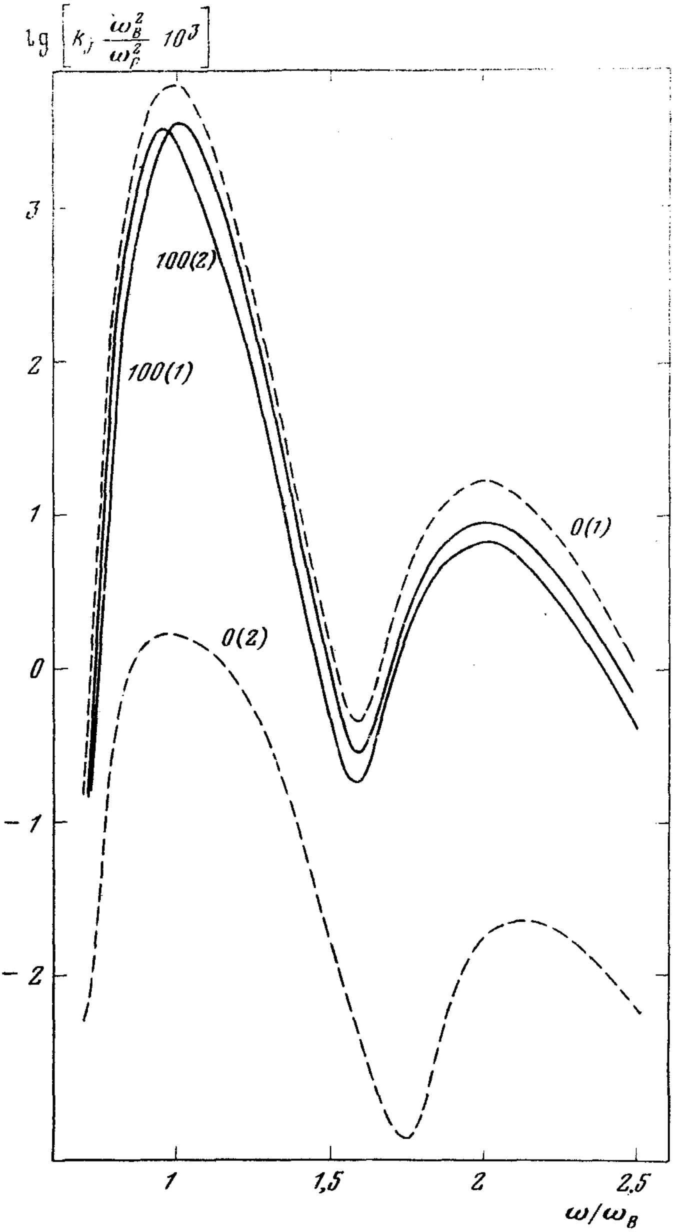

FIG. 3. Coefficients k_j for the first two resonances at $\theta = 30°$.

$\gg 1$ the ellipticity decreases and the behavior of $K_j(\omega)$, $\chi_j(\omega)$ becomes nonmonotonic. Intersection points of K_j, χ_j appear in the vicinity of resonance the frequencies ω^*, which are determined from the condition $q = \mathrm{Re}(n_L/n_C) = 0$. Upon passage through ω^* there occurs either a 90° rotation of the positional angles (for $|p| = |\mathrm{Im}(n_L/n_C| < 1)$ or a change in the sign of the circular polarization (for $|p| > 1$). For $|p| = 1, q = 0$ $[\theta \approx \pi^{1/4}(\beta V)^{-1/2}$ for $\beta V \gg 1]$ a complete coincidence (maximum nonorthogonality) of the NW polarization occurs ($K_j = 0, \chi_j = -45 \sin p$). Let us recall that in a vacuum-free plasma $|p| \ll |q|$, and the NW polarizations are practically orthogonal. For $V \sin^2 \theta \gg 1$ the NW polarization become linear (as in a pure vacuum) and orthogonal.

b) The change in the NW polarization because of vacuum polarization alters the ratio between the NW absorption coefficients. In a rarefied vacuum-free plasma at the first cyclotron resonance in a dipole approximation the absorption (emission) of only (extraordinary) NW is allowed. The absorption of an ordinary NW, associated with the contribution of higher multipoles, is less by a factor of $\sim \beta^{-2}$. A change in NW polarization alters the selection rules. As a result, when $V\beta |\cos \theta| \gg 1$, the resonance absorption coefficients of both NW are comparable in order of magnitude (Figs. 2, 3). For $V\beta |\cos \theta| \gg 1$, $V \gg \tan^2 \theta$, $V \sin \theta \gg 1$

$$k_1 \approx \frac{\sqrt{\pi}\,\omega_p^2}{4\omega^2 \beta |\cos \theta|} \exp(-z_1^2), \quad k_2 \approx k_1 \cos^2 \theta, \quad z_l = \frac{\omega - l\omega_B}{\omega\beta \cos \theta} \tag{4}$$

For $V\beta |\cos \theta| \ll 1, z_1 \lesssim 1$

$$k_1 \approx \frac{\sqrt{\pi}\,\omega_p^2 \exp(-z_1^2)}{4\omega^2 \beta |\cos \theta|} (1 + \cos^2 \theta),$$

$$k_2 = \beta^2 \frac{k_1}{\pi} \left(\frac{\sin\nu \cos\nu}{1 + \cos^2\nu} \right)^4 \frac{(3 + \tan^2\nu - 2V\omega^2\omega_B^{-2})^2}{\exp(-2Z_1^2) + 4\pi^{-1} Z_1^2}, \tag{5}$$

that is for $V > 3 + \tan^2 \theta$, vacuum polarization enhances the absorption of the ordinary wave, while for $V \approx (3 + \tan^2 \theta)\,\omega_B^2/(2\omega^2)$ it suppresses it; as a result a double resonance appears in k_2 (Fig. 2). Since the vacuum does not absorb photons for $h\omega < 2mc^2$, $B^2 \ll B_c$, then $k_1 + k_2$ does not depend on V.

c) For $V = 0$ the NW absorption maximums are displaced from $z_1 = 0$ by the small amount $\delta z_1 \lesssim \beta \cos \theta$, different for different NW. For $1 \lesssim V \sin^2 \theta \lesssim \beta^{-2}$ the shifts are increased as they reach the maximum value $\delta z_{\max} \sim \cos \theta$ of the order of the Doppler resonance width when $V\beta \sin^2 \theta \sim 1$.

d)Vacuum polarization shifts the intersection point of $\kappa_j(\omega)$ and can lead to intersections of k_j and κ_j occurring for $\theta \approx \pi^{1/4}(\beta V)^{-1/2}$. With an increase in V the absorption coefficient of one of the NW increases while the other decreases, so that a reverse change in the ratio between k_1 and k_2 is possible.

e) An absorption increase in the ordinary NW and a reduction in the extraordinary occur at the higher cyclotron harmonics ($l \geqslant 2$) within a broad interval of angles (Fig. 3);

$$k_j = \frac{\sqrt{\pi}\,\omega_p^2 \beta^{2l-3} l^{2l} (\sin \theta)^{2l-2}}{2^{2l+1} \omega^2 l! |\cos \theta|} e^{-z_l^2} \left[1 + \cos^2 \theta \pm \frac{2 \cos \theta + q \sin^2 \theta}{\sqrt{1 + q^2}} \right],$$

$$q = \frac{\sin^2 \theta}{2l \cos \theta} [1 + V(l^2 - 1)]. \tag{6}$$

4. Vacuum polarization drastically alters the polarization of the cyclotron emission of both an optically thin as well as thick plasma. For $V \gg 1$, by and large the circular polarization is suppresed and the linear is increased (for the higher harmonics). The degree of linear polarization of the radiation of an optically thin plasma for $\omega \approx \omega_B$ changes little. For $V\beta\cos\theta \lesssim 1$ near the points ω_* the circular polarization can increase. At the intersection points of n_j a strong transformation from one NW to the other occurs. At the intersection points of k_j the cyclotron radiation is depolarized. In the resonance region $\omega \approx \omega_B$ the polarization can depend on the frequency nonmonotonically.

The cyclotron radiation spectrum of an optically thin plasma is not altered. For the optically thick plasma in the emitting region of x-ray pulsars vacuum polarization contributes to an enhancement of the cyclotron lines. The enhancement is accompanied by an additional shift, a broadening and a more complicated form of contour lines. The vacuum polarization effects are strongly dependent on θ and alter the angular distribution of the cyclotron radiation.

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