

# Instability of homogeneous precession of magnetization in the superfluid A phase of He<sup>3</sup>

I. A. Fomin

*L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences*

(Submitted 20 June 1979)

*Pis'ma Zh. Eksp. Teor. Fiz.* **30**, No. 3, 179–181 (5 August 1979)

We show that in a strong magnetic field the spatially homogeneous precession of magnetization in the A phase of He<sup>3</sup> is unstable with respect to the excitation of spin waves. We calculate the time for longitudinal and transverse relaxation due to the development of the instability.

PACS numbers: 67.50.Fi, 76.60.Es

The spatially homogeneous precession of magnetization is usually considered in a theoretical description of pulsed NMR experiments for He<sup>3</sup> superfluid phases. However, as indicated by experiment,<sup>[1]</sup> the de-phasing time  $\tau_2$  in these phases is usually less or of the order of the longitudinal relaxation time  $\tau_1$ . In this connection there arises a question concerning both the reason for the de-phasing and also the possible effect it has on the magnetization relaxation process.

Equations for the spin dynamics in the inhomogeneous case are obtained by augmenting Leggett's equations<sup>[2]</sup> with terms arising from the spatial rigidity of the condensate and from spin diffusion. Here we will consider the case of strong magnetic fields  $\mathbf{H}_0$  such that the Zeeman energy  $g(\mathbf{S}, \mathbf{H}_0)$  ( $\mathbf{S}$  is the spin, and  $g$  is the gyromagnetic ratio for He<sup>3</sup>) is large in comparison with the spin-orbital energy  $V_A$  and with the nonuniform condensate energy  $F$ . In these fields, according to the approach in Ref. 3,  $V_A$  and  $F$  should be averaged over quantities which vary with frequencies on the order of the Larmor frequency  $\omega_L$ . As a result, for the A-phase

$$F = \frac{c^2}{2\omega_L^2} \left\{ \frac{(1-u)(3-u)}{2} (\partial_1 \alpha)^2 + \frac{1}{2} (\partial_1 \beta)^2 + (\partial_1 \Phi)^2 - 2(1-u)(\partial_1 \alpha)(\partial_1 \Phi) \right\}. \quad (1)$$

Here  $u = \cos \beta$ ,  $\Phi = \alpha + \gamma$ ;  $\alpha, \beta, \gamma$  are the Euler angles describing the rotation of the spin part of the order parameter from its initial position,  $(\partial, \alpha)^2 = 2(\nabla \alpha)^2 - [(1, \nabla) \alpha]^2$ ,  $\mathbf{1}$  is a vector characterizing the orientation of the orbital part of the order parameter, and  $c$  is a constant having the sense of a spin wave velocity. Near the transition temperature  $T_c$ ,  $c^2 \sim v_F^2(1 - T/T_c)$ , where  $v_F$  is the Fermi speed.

$$V_A = - \frac{\Omega_A^2}{8\omega_L^2} \left[ u^2 + \frac{(1+u)^2}{2} \cos 2\Phi \right]. \quad (2)$$

Here  $\Omega_A$  is the frequency of the longitudinal oscillations. Both  $V_A$  and  $F$  have their measurement lost by division into the Zeeman energy in the equilibrium state. The averaged  $V_A$  and  $F$  are independent of the angle  $\alpha$ , and as a result their corresponding equations of motion have the solution with  $\nabla\alpha = \mathbf{h} = \text{const}$ ,  $\nabla\beta = \nabla\Phi = 0$ . This solution describes the precession of a magnetization spiral occurring with a frequency

$$\dot{\alpha} = -\omega_L \left\{ 1 - \frac{\partial V_A}{\partial u} + \frac{c^2}{2\omega_L^2} (2-u) [2h^2 - (\mathbf{h}, \mathbf{l})^2] \right\}. \quad (3)$$

For  $\beta \rightarrow 0$  this equation becomes the dispersion law for transverse spin waves<sup>(4)</sup> in the limit of large  $\omega_L$ , i.e., the resulting solution corresponds to spin waves of large amplitude. Linearization of the equations of motion about this solution shows that for not too large  $h$  (neglecting diffusion for  $h \lesssim \Omega/c$ ) disturbances of the form  $\exp[i(\mathbf{k}\mathbf{r} - \omega t)]$  will increase. This instability is due to the spin-orbit interaction and is analogous to the Suhl instability in ferromagnetics.<sup>(5)</sup> For uniform precession ( $h = 0$ ) the dispersion equation for the perturbations has the following form:

$$(\omega - iDk^2) = \frac{c^2}{2} (1-u)(3-u) \langle \mathbf{k}, \mathbf{l} \rangle \widetilde{V}_{uu} - (1-u^2) \frac{c^4}{\omega_L^2} \langle \mathbf{k}, \mathbf{l} \rangle^2 \frac{\widetilde{V}_{uu}}{\widetilde{V}_{\Phi\Phi}}, \quad (4)$$

where

$$\langle \mathbf{k}, \mathbf{l} \rangle = 2k^2 - (\mathbf{k}, \mathbf{l})^2, \quad \widetilde{V}_{uu} = -\frac{3\Omega_A^2}{8\omega_L^2} + \frac{c^2 \langle \mathbf{k}, \mathbf{l} \rangle}{2(1-u^2)\omega_L^2},$$

$$\widetilde{V}_{\Phi\Phi} = \frac{\Omega_A^2}{4\omega_L^2} (1+u)^2 + \frac{c^2}{\omega_L^2} \langle \mathbf{k}, \mathbf{l} \rangle,$$

and  $D$  is the spin diffusion coefficient. Diffusion turns out to be important since the  $A$ -phase occurs only for temperatures close to  $T_c$ . When  $T \rightarrow T_c$ ,  $c^2$  and  $\Omega_A^2 \rightarrow 0$ , and  $D$  tends to a finite value. For magnetization deviation angles  $\beta \sim 1$  the perturbations with  $k = k_{\max} \sim \Omega_A c / D\omega_L$  increase most rapidly with a corresponding growth increment  $\Gamma \sim \Omega_A^2 c^2 / D\omega_L^2$ . In the neighborhood of the angles  $\beta = 0$  and  $\beta = \pi$  the instability is displaced to the region of small  $k$  by the second term in the equation for  $\widetilde{V}_{uu}$ . As a result, the development of the instability is slowed, and this slowing down occurs for  $\sin \beta \lesssim c^2 / D\omega_L$ .

As long as the development of the instability begins with a nonzero value for  $k$ , it should be expected that as result of the loss of stability the spin system enters a turbulent state characterized by large fluctuations in the magnetization density. The time for the development of fluctuation determines the de-phasing time  $\tau_2 \sim \Gamma^{-1}$  to an order of magnitude. Together with spin diffusion, the fluctuations are an effective mechanism for longitudinal relaxation. To evaluate  $\tau_1$  we note that the spin diffusion leads to a change in the energy according to

$$\frac{dE}{dt} = - \frac{Dg^2}{\chi} \frac{dS_i}{dx_k} \frac{dS_i}{dx_k}, \quad (5)$$

where  $\chi$  is the paramagnetic susceptibility, and the line denotes an average over the fluctuations. According to the formula given,  $\tau_1 \sim Dk_{\max}^2 \sim \Gamma^{-1} \sim \tau_2$ . The numerical values lead to times which are less than those actually observed. As for normal turbulence, however, it should be kept in mind that the development of instability will be inhibited if there are no initial perturbations in the system.

According to the calculation above, for  $T \rightarrow T_c$  when  $\tau_1$  and  $\tau_2$  go to infinity as  $(1 - T/T_c)^{-2}$ , i.e., faster than  $\tau_1$  for the "internal" relaxation mechanism of Leggett and Takagi,<sup>[6]</sup> and in close proximity to  $T_c$  the latter becomes more effective. The relaxation times corresponding to these two mechanisms are compared at  $\Omega_A^2 \bar{\tau} \ln(\alpha_0) \sim (\Delta/\Delta_0)^3$ . Here  $\bar{\tau}$  is the time between quasiparticle collisions,  $\Delta$  is the spacing in the perturbation spectrum,  $\Delta_0$  is the value for the spacing at  $T = 0$ , and  $\alpha_0$  is the amplitude of the initial perturbations of the angle  $\alpha$  with  $k \approx k_{\max}$ . Substitution of numbers leads to the value  $(1 - T/T_c) \sim 10^{-3}$  for the temperature at which the times are compared.

In addition to the transverse mode for the spin waves discussed here, there is another which at  $k = 0$  becomes longitudinal magnetization oscillations. This mode is stable and has no effect on the relaxation. As shown in research done earlier,<sup>[7]</sup> in the  $B$  phase of the Leggett configuration homogeneous precession is stable; however, instability may arise in the structure.

<sup>1</sup>L.R. Corruccini and D.D. Osheroff, Phys. Rev. B-17, 126 (1978).

<sup>2</sup>A.J. Leggett, Ann. Phys. 85, 11 (1974).

<sup>3</sup>I.A. Fomin, J. Low. Phys. 31, 509 (1978).

<sup>4</sup>A.J. Leggett, Rev. Mod. Phys. 47, 331 (1975).

<sup>5</sup>H. Suhl, Phys. Chem. Sol. 1, 209 (1957).

<sup>6</sup>A.J. Leggett and S. Takagi, Ann. Phys. 106, 79 (1977).

<sup>7</sup>I.A. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. 28, 362 (1978) [JETP Lett. 28, 334 (1978)].