Energy-independent phase-shift analysis of the $\pi^-\pi^0$ interaction from the reaction $\pi^-p \to \pi^-\pi^0 p$

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Results are presented of a phase-shift analysis of $\pi^-\pi^0$ scattering in the interval $360 < m_{\pi\pi} < 1300$ MeV, obtained by an energy-independent method and with averaged spherical harmonics extrapolated to the pion pole. The importance of taking interference with higher waves into account is demonstrated with the D_2 wave as an example.

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For a phase-shift analysis of $\pi\pi$ scattering it is customary to choose the states $\pi^-\pi^+$ or $\pi^\pm\pi^\pm$. The first is attractive in having a complete set of the phase shift, including the interesting S_0 wave and the D-wave f^0 resonance, states with a pair of identically charged pions—the relative simplicity of the analysis because of the presence of only waves with isospin $I=2$. The $\pi^\mp\pi^0$ states do not enjoy great popularity, and the only published paper known to the authors is[1]. The apparent reason is the presence of an "extra" (compared with the $\pi^\pm\pi^\pm$ states) P wave with a clearly pronounced resonance, which makes it difficult to obtain the phase shifts δ_0^2 and δ_2^2. There exists, however, a problem in which this shortcoming of the $\pi^\pm\pi^0$ state turns into an advantage. The point is that in the phase-shift analysis of such states it is possible not only to obtain simultaneously the phase shifts δ_1^1 and δ_0^2 and to study the influence of contributions of higher waves with the D_2-wave as an example, but also to examine the S—P and P—D interferences. It is known that the contribution of higher waves in the region of small dipion masses is frequently neglected in phase-shift analysis. One can expect, however, the interference with higher waves to be large enough to make this neglect unjustified. The present paper is devoted to these questions.

The work was performed using the statistical material of ~5000 events of the reaction $\pi^-p \to \pi^-\pi^0 p$, obtained in a liquid-hydrogen bubble chamber at an

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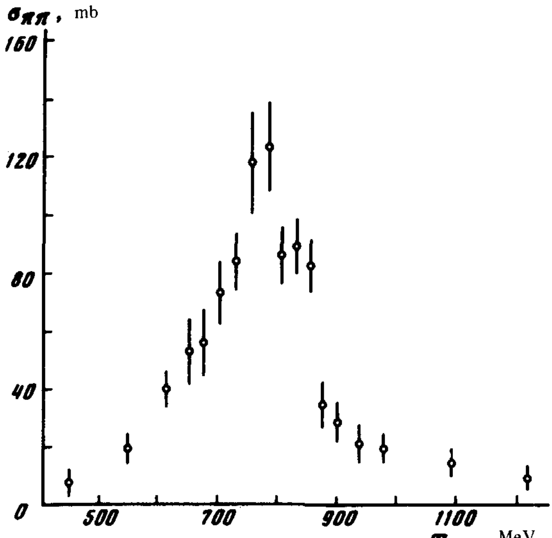

FIG. 1.

incidence-pion momentum 4.5 GeV/c. The phase-shift analysis was carried out by an energy-independent method, i.e., in each interval separately, in the region of dipion masses $380 < m_{\pi^-\pi^0} < 1300$ MeV. The experimental data were

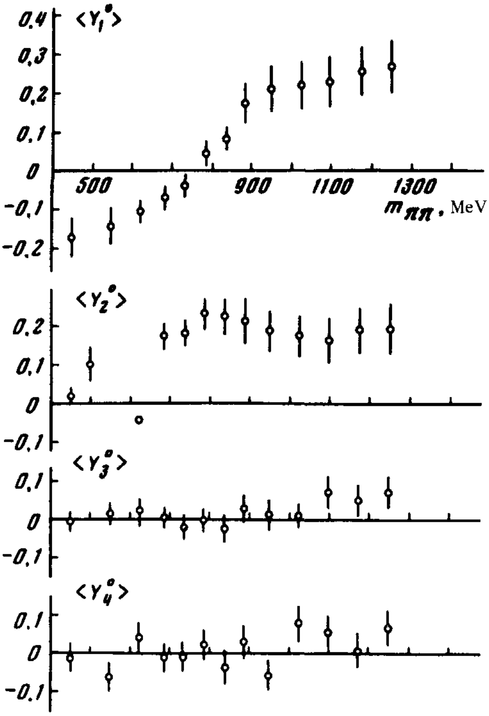

FIG. 2.

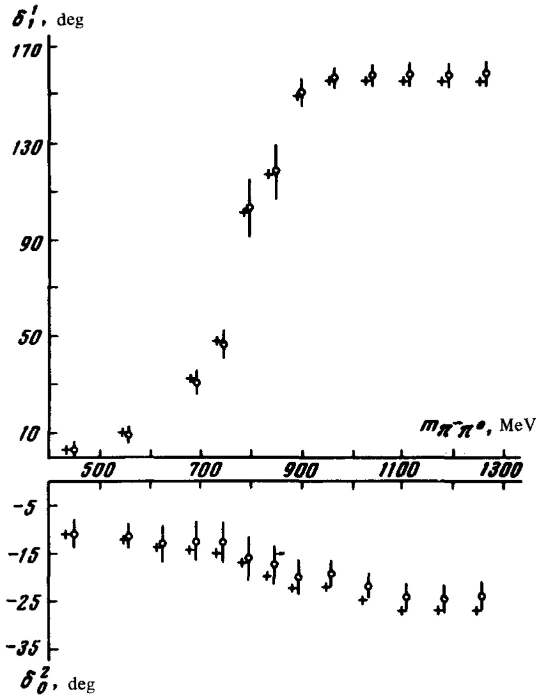

FIG. 3.

the pion cross sections $\sigma_{\pi^-\pi^0}$ and the averaged spherical harmonics $\langle Y_L^0 \rangle$ $> (t, m_{\pi\pi})$ extrapolated to the pion hole. The averaged harmonics were defined as

$$< Y_L^0 > = \frac{\sum_{i=1}^{N} Y_L^0(\theta_i)}{N}$$

The cross sections $\sigma_{\pi^-\pi^0}$ were obtained by the Chew-Low method, and in the extrapolation we used both a linear law and a quadratic law. The choice between these possibilities was based on the χ^2 criterion. Figures 1 and 2 show the obtained cross sections of the $\pi^-\pi^0$ interaction and the extrapolated average harmonics $\langle Y_L^0 \rangle$ for $L = 1, \ldots, 4$. Some qualitative information can be obtained directly from the outward appearance of the harmonics. In the case of elastic interactions, assuming the amplitude $f_l^I = \sin\delta_l^I e^{i\delta_l^I}$, the expressions for the spherical harmonics can be written in explicit form in the following manner:

$$< Y_1^0 > = \frac{4\pi \lambda\!\!\!^{-2}}{\sigma_{\pi^-\pi^0}} \left\{ \sqrt{\frac{\pi}{3}} \sin\delta^1 \sin\delta_0^2 \cos(\delta_1^1 - \delta_0^2) + 2\sqrt{\frac{\pi}{3}} \sin\sigma_1^1 \sin\delta_2^2 \times \cos(\delta_2^2 - \delta_1^1) \right\},$$

$$\langle Y_2^0 \rangle = \frac{4\pi\lambda^2}{\sigma_{\pi^-\pi^0}}\left\{\frac{3}{\sqrt{5\pi}}\sin^2\delta_1^1 + \sqrt{\frac{5}{\pi}}\sin\delta_0^2\sin\delta_2^2\cos(\delta_0^2-\delta_2^2) + \frac{5}{7}\sqrt{\frac{5}{\pi}}\sin^2\delta_2^2\right\},$$

$$\langle Y_3^0 \rangle = \frac{4\pi\lambda^2}{\sigma_{\pi^-\pi^0}}\left\{\frac{9}{\sqrt{7\pi}}\sin\delta_1^1\sin\delta_2^2\cos(\delta_1^1-\delta_2^2)\right\}, \qquad (1)$$

$$\langle Y_4^0 \rangle = \frac{4\pi\lambda^2}{\sigma_{\pi^-\pi^0}}\left\{\frac{15}{7\sqrt{\pi}}\sin^2\delta_2^2\right\}.$$

It is seen therefore that the reversal of the sign in $\langle Y_1^0\rangle$ is due to the term with $\cos(\delta_1^1-\delta_0^2)$, which reverses sign when $(\delta_1^1-\delta_0^2)$ goes through 90° (the second term is small). From the fact that $\delta_1^1=90°$ at $m_{\pi\pi}=m_\rho$ and that δ_0^2 is negative, it is clear that the passage through zero should occur at $m_{\pi\pi}$ somewhat smaller than the mass of the ρ^- resonance. It is seen from Fig. 2 that the reversal of the sign takes place at $m_{\pi\pi}\sim 750$ MeV, in agreement with the position of the ρ^- resonance. $\langle Y_2^0\rangle$ increases from the small values due to the prevailing of the S_2 wave over P_1 at small $m_{\pi\pi}$ to a maximum in the region of the ρ^- resonance, after which it remains practically constant and showing a weak change of the phase shifts δ_0^1 and δ_0^2 in the region above 1000 MeV. In the entire region, $\langle Y_2^0\rangle$ remains positive, as expected.[1)] Both $\langle Y_3^0\rangle$ and $\langle Y_4^0\rangle$ oscillate near zero up to $m_{\pi\pi}\sim 1100$ MeV, and then become weakly positive. This demonstrates the smallness of the D_2 wave in the investigated region. Unfortunately, owing to the large sensitivity of $\langle Y_3^0\rangle$ and $\langle Y_4^0\rangle$ to statistical fluctuations, it is difficult in our statistics to use these harmonics to obtain the phase shift δ_2^2. We therefore calculated only the phase shifts δ_1^1 and δ_0^2, and took the values of δ_2^2 from[2]. We used in the calculations only $\langle Y_1^0\rangle$ and $\langle Y_2^0\rangle$, and the system was solved by the iteration method.

Figure 3 shows the obtained phase shifts δ_1^1 and δ_0^2. The crosses mark the phase shifts obtained in the absence of the D_2 wave, and the points are the results with D_2 taken into account. It is seen that the D_2 wave exerts little influence on the phase shift δ_1^1 in the region of small dipion masses, and its influence is more noticeable at $m_{\pi\pi}>900$ MeV. Allowance for the D_2 wave decreases δ_1^1 up to $m_{\pi\pi}=m_\rho$, and increases it at $m_{\pi\pi}>m_\rho$. The influence of the D_2 wave on S_2 is more noticeable, aided also by the smaller value of S_2. The difference in the results obtained with and without allowance for the D_2 wave amounts at all times $\sim$15–20% of the value of the phase shift. Allowance for D_2 leads to a decrease in the absolute values of the δ_0^2 phase shifts in almost the entire region of $m_{\pi\pi}$ (with the exception of a small region near $m_{\pi\pi}$). It should be noted that the influence of the D_2 wave itself, on both δ_1^1 and δ_0^2, is very small and practically all the changes are due to interference terms. The S_2–D_2 and P_1–D_2 interferences turn out to be far from negligible even at $m_{\pi\pi}<900$ MeV, and must be taken into account in practically the entire region. It should be emphasized that the quantitative changes in δ_2^2 have little effect on the results, what is important is the very fact of allowing or not allowing for the interference terms.

In conclusion, the authors thank I. I. Gurevich for a discussion of the results.

1) The value of $\langle Y_2^0 \rangle$ at $m_{\pi\pi} = 625$ MeV is statistically an aberration and was not used in the calculation.

[1] J. Baton, G. Laurens, and J. Reignier, Phys. Lett. 25B, 419 (1967).
[2] W. Hoogland, B. Hyams *et al.*, Nucl. Phys. B **69**, 266 (1974).