Numerical investigation of the collapse of Langmuir waves in a magnetic field

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The existence of a collapse of Langmuir waves in a magnetic field, for an initial electric-field distribution in the form of a dipole caviton, is established on the basis of a numerical experiment.

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The dynamics of the collapse of a dipole packet of Langmuir waves in the absence of a magnetic field has by now been investigated in detail for the planar and axially-symmetrical cases.[1–3] It has been shown that the collapse has an isotropic self-similar regime. In the case of collapse in a magnetic field, there is only an asymptotic solution that points to an anisotropic collapse regime.[4] The present paper is devoted to a numerical investigation of the collapse of Langmuir waves in a magnetic field in an axially symmetrical dipole model.

Following[1–5], we write down in dimensionless form the fundamental system of equations with allowance for a magnetic field directed along the z axis

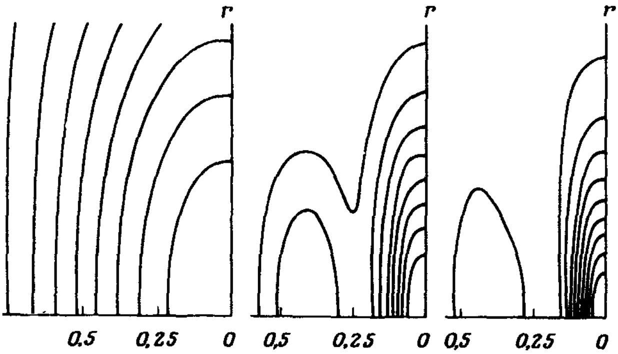
$$2i\rho_t + \Delta\rho + A\,\Delta_\perp\Psi = \nabla(\eta\,\nabla\Psi), \quad \Delta\Psi = \rho \tag{1}$$

$$\eta_{tt} - \Delta\eta = \Delta|\nabla\Psi|^2, \quad \Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r},$$

where ρ and Ψ are the complex charge and electrostatic potential of the field, and η is the variation of the plasma concentration. The transition from dimensional variables to dimensionless ones is determined by the formulas:

$$t = \frac{3M}{m\omega_p}t', \quad \mathbf{r} = 3\sqrt{\frac{M}{m}}\lambda_D\mathbf{r}', \quad A = 3\frac{M}{m}\frac{\omega_H^2}{\omega_p^2} \tag{2}$$

$$\delta n = \frac{m}{3M}n_0\eta, \quad |\nabla\Psi|^2 = \frac{16\pi m}{3M}n_0T_e|\nabla\Psi'|^2,$$



IG. 1. Level lines of $|\mathbf{E}|^2$ in the regime $A=100$ and $\rho_0=19$, for $t=0$, 0.125, and 0.175; $|\mathbf{E}|^2_{max}$ =97.4, 384, and 1364.

'here ω_p is the electron plasma frequency, λ_D is the Debye radius, n_0 is the unperturbed plasma oncentration, T_e is the electron temperature, and ω_H is the electron cyclotron frequency. The ystem (1) has conservation integrals:

$$I_1 = \int |\nabla\Psi|^2 d\mathbf{r} \qquad \text{(number of plasmons)} \tag{3}$$

$$I_2 = \int \left\{ \frac{V^2}{2} + \frac{\eta^2}{2} + |\rho|^2 + \eta\,|\nabla\Psi|^2 + A\,|\nabla_\perp\Psi|^2 \right\} d\mathbf{r} \tag{4}$$

here $\mathbf{V}$ is the macroscopic plasma velocity.

At the initial instant of time $t=0$, the charge distribution in a cylinder ($|z|\leqslant\pi$, $r\leqslant\pi$) is specified s follows:

$$\rho = \begin{cases} \rho_0\sqrt{\omega}\,\sin\dfrac{\pi z}{2}, & \omega \geqslant 0 \\ 0, & \omega < 0 \end{cases} \qquad \omega = 1 - \frac{r^2+z^2}{4}. \tag{5}$$

n addition, it is assumed that $t=0$

$$\eta = -|\nabla\Psi|^2, \qquad \eta_t = 0. \tag{6}$$

t the boundaries $z=\pm\pi$ and $r=\pi$ of the region covered by the calculation, we impose homogen- ous boundary conditions of the second kind

$$\frac{\partial}{\partial n}\rho = \frac{\partial}{\partial n}\Psi = \frac{\partial}{\partial n}\eta = 0.$$

These conditions define an anisotropic dipole cavern with axes along the magnetic field, with I_1 and I_2 conserved in the cylinder under consideration, and with the transverse cavern dimension much larger than the longitudinal direction $l_\parallel$.

A numerical calculation of the evolution of the initial distribution of the field and of the lasma concentration was carried out in the parameter range:

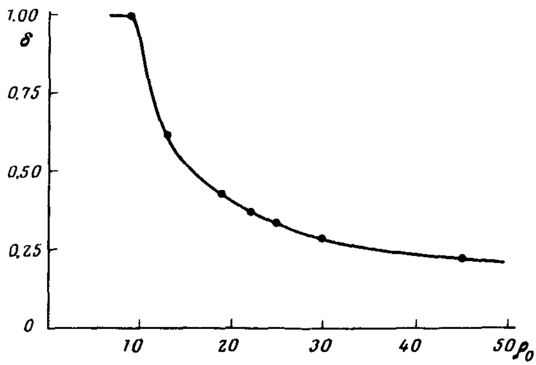


FIG. 2. Dependence of the relative frac tion δ of the plasmons in the principa collapsing cavern on the initial charge ρ_0

$A=0$–500; $\rho_0=4$–45.

Just as in the absence of a magnetic field, the collapse condition is $I_2<0$. The appearance in *1* of a large positive term connected with the magnetic field causes the collapse to develop a substantially higher initial charge densities ρ_0.

Let us examine in greater detail the regime $A=100$, $\rho_0=19$. The calculated distribution of th $|\mathbf{E}|^2$ level lines, corresponding to a dipole cavern, has several maxima (3 at $\rho_0=19$). During th evolution of the modulation instability, the central collapsing caviton, which corresponds to th principal maximum of $|\mathbf{E}|^2$, captures plasmons from the peripheral region. It turned out here tha at small initial charges ($\rho_0\leqslant 9$) practically the entire energy goes into the collapse, and witl increasing ρ_0 the fraction of the plasmons in the central collapsing cavern decreases (this result i obtained also in the absence of a magnetic field and can be noticed on the plots for $\rho_0=13$[1]). Th dependence of the relative fraction δ of the plasmons in the central collapsing caviton on ρ_0 is give in Fig. 2. It should be noted that under conditions when a small fraction of the plasmons collapse in the central caviton, a collapse can develop also in the peripheral maxima of the field.

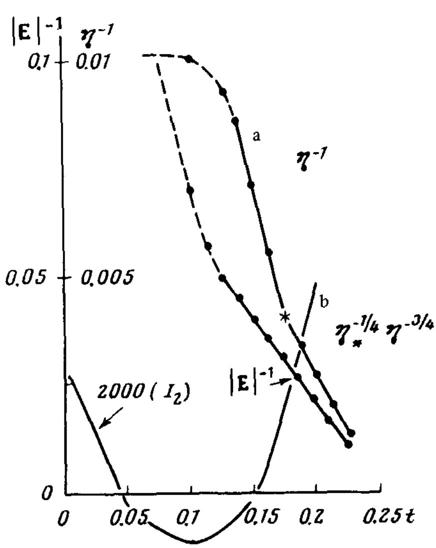


FIG. 3. Self-similarity regimes at $A=100$, $\rho_0=19$: a—anisotropic, b—isotropic; *—point where self-similarity changes, the caviton dimensions are determined in the section $|\mathbf{E}|^2=0.5|\mathbf{E}|^2_{\max}$.

To compare the numerical solutions with the asymptotic self-similar solutions[4] we checked ı the self-similarity of the variation of the charcteristic dimensions of the caviton, and of the field ıd density increases. It follows from the results that during the initial stage, when the influence of e magnetic field is substantial, the transverse dimension of the cavern varies more slowly than the ngitudinal one $l_\perp \sim l_\parallel^2 \sim t_0 - t$, in agreement with[4]. The laws governing the growth of the field and ˈ the density during the initial stage of the collapse of caviton $|\mathbf{E}|^{-1} \sim \eta^{-1} \sim t_0 - t$ (Fig. 3) also ;ree with[4]. In the course of the collapse, the influence of the magnetic field becomes negligible ıd the laws governing the variation of the dimensions, the field, and the density follow those of an ɔtropic caviton $l_\perp \sim l_\parallel \sim |\mathbf{E}|^{-1} \sim \eta^{-3/4} \sim t_0 - t$. In this regime, the value of I_2 amounts on the ˈerage to 5% of the terms in (4), and the variation of I_2 is shown in Fig. 3.

Thus, the obtained numerical solutions confirm the existence of a collapse of Langmuir waves a magnetic field. Although the magnetic field hinders the formation of the collapse (the integral increases), the dipole distribution of the electric field must without fail go into the collapse gime at $I_2 < 0$. It appears that a stationary solution in the form of a three-dimensional soliton[6] is ɔt realized in this case.

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