›pearance of combination frequencies in the ıubnikov–de Haas oscillations and in oscillations the surface impedance of tellurium-doped Bi ıd $Bi_{1-x}Sb_x$ alloys

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Strong combination frequencies (with amplitude on the order of the amplitude of the fundamental harmonics) have been observed in the Shubnikov–de Haas oscillations and the surface-impedance oscillations of tellurium-doped Bi and $Bi_{1-x}Sb_x$. These combination frequencies were not observed in the de Haas–van Alphen oscillations of the same alloys under analogous conditions.

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1. The change of the state density $n(\epsilon)$ on the Fermi level ϵ_F in a magnetic field H causes the ponents of the conductivity tensor $\hat{\sigma}_{\alpha\beta}$ to have, besides a monotonic component σ_0, also lating increments $\Delta\sigma_1$ and $\Delta\sigma_2$

$$\hat{\sigma}_{\alpha\beta} = (\hat{\sigma}_0)_{\alpha\beta} + (\Delta\hat{\sigma}_1)_{\alpha\beta} + (\Delta\hat{\sigma}_2)_{\alpha\beta}. \quad (1)$$

The quantum correction $\Delta\sigma_1$ constitutes a contribution to the conduction from the scattering ı transitions between different Landau levels $(N+1/2)\hbar\omega$ $(\Delta N\neq 0)$, while $\Delta\sigma_2$ describes the :ribution due to scattering within the Landau level closest to ϵ_F $(\Delta N=0)$. It was shown in[1] the amplitudes $\Delta\sigma_1$ and $\Delta\sigma_2$ have the following dependences on the monotonic $[n_0(\epsilon)]$ and llating $[n_1(\epsilon)]$ parts of the state density and the relaxation time τ:

$$\Delta\sigma_1 \sim n_0(\epsilon) n_1(\epsilon) \tau^{1/2}; \quad \Delta\sigma_2 \sim [n_1(\epsilon)]^2 \tau . \quad (2)$$

For large quantum numbers $(N \gg 1)$, $\Delta\sigma_1 \gg \Delta\sigma_2$ and is a linear function of the oscillations of magnetic moment M at arbitrary shape of the Fermi surface and at an arbitrary charactei of scattering[2]: $\Delta\sigma_1 \sim \partial M/\partial H$. In this case the oscillations of the magnetoresistance $\partial\rho/\partial H$ and of surface impedance $\partial Z/\partial H$ can be described in terms of the oscillations of ΔM[3,4]:

TABLE I.

Observed frequencies ω_i (rel. un.)	Statistical weight of $I(\omega)$ (rel. un.)	Combination of frequencies $n\omega_1 + m\omega_2$
11	1.00	$\omega_1 = 11$ (section S_1)
22	0.62	$2\omega_1 = 22$
27	0.92	$\omega_2 = 27$ (section S_2)
32	0.25	$3\omega_1 = 33$
38	0.46	$\omega_1 + \omega_2 = 38$
49	0.25	$2\omega_1 + \omega_2 = 49$
60	0.30	$3\omega_1 + \omega_2 = 60$
72	0.14	$4\omega_1 + \omega_2 = 71$

$$\Delta\rho \sim \rho_0 \sum_{r=1}^{\infty} a_r \sin\left(\frac{2\pi\epsilon_F}{\hbar\omega_c} - \frac{\pi}{4}\right).$$

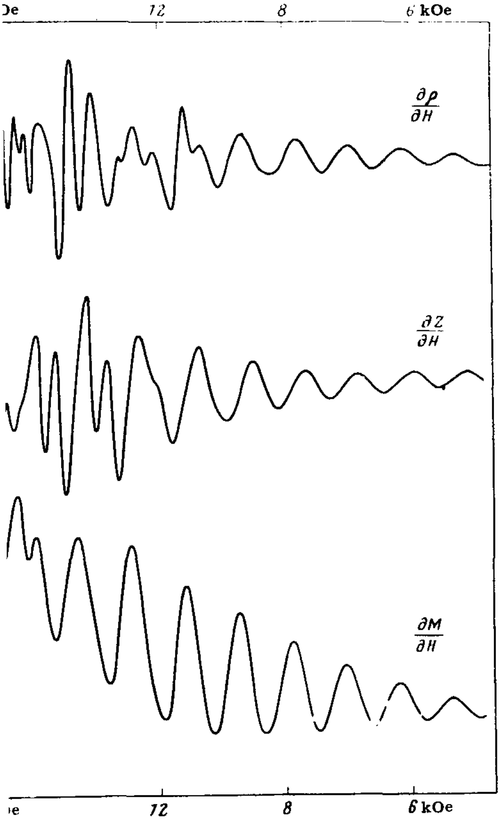
Here a_r is the amplitude of the rth harmonic. For not too low temperatures we can cor ourselves in (3) to the term $r=1$.

In the presence of several extremal sections (or several equal-energy surfaces), the oscillat of $\Delta\rho(H)$ ($\Delta Z(H)$,$\Delta M(H)$) are superpositions of the frequencies ω_i and their harmonics $n\omega_i$, w are proportional to the corresponding extremal sections and whose contribution to the oscillat is usually small ($\sim$5%).

2. We present here the results of an investigation of Shubnikov—de Haas and de Haas—Alphen oscillations and of the surface impedance of tellurium-doped (0.01 at.%)Bi and Bi_{1-} alloys at liquid-helium temperature.

Figure 1 shows plots of $\partial\rho/\partial H$, $\partial Z/\partial H$ and $\partial M/\partial H$ against the reciprocal magnetic field for the Bi-Te alloy with the field oriented along the bisector axis corresponding to two extre sections S_1 and S_2. It is seen that the $\partial\rho/\partial H$ and $\partial Z/\partial H$ curves differ qualitatively from oscillations of $\partial M/\partial H$ (this difference is observed only in doped alloys, while for Bi all the p are similar in character). The plots of the spectral density $I(\omega)$ of the $\partial M/\partial H$ oscillation Bi+Te (Fig. 2, curve 3) reveals clearly two peaks corresponding to the sections S_1 and S_2 of electron part of the Fermi surface. The harmonic content of the oscillations of $\partial\rho/\partial H$ and ∂Z is much more complicated (Fig. 2, curves 1 and 2). Besides the fundamental frequencies ω_1 anc one can see clearly peaks of ω_i which do not correspond to the extremal sections S_1 and S_2 and well described by the formula $\omega_i = n\omega_1 + m\omega_2$ (n and m are integers). Table I lists the results Fourier analysis of the $\partial\rho/\partial H$ oscillations in Bi-Te at a specially selected field orientation makes it possible to distinguish between the frequency combinations and the harmonics.

A characteristic feature of the results, besides the appearance of combination frequencie the abrupt increase (compared with undoped Bi) of the amplitude of the second harmonics o fundamental frequency. In individual cases this effect becomes so strong that what is observe practice is only the second harmonic. This situation takes place in semiconducting $Bi_{1-x}Sb_x$ al doped with Te. The frequency doubling is particularly pronounced in the investigation of l



. 1. Plots of the derivatives of the magnetoresistance ($\partial\rho/\partial H$), of the surface impedance ∂H), and of the magnetic moment ($\partial M/\partial H$) against the reciprocal magnetic field $1/H$ for the Bi-lloy. The field H is parallel to the bisector axis.

ons of the electron Fermi surface (Fig. 3). At angles $|\phi|>20°$, the fundamental frequency ominates, while at $|\phi|<15°$ the fourth harmonic appears (the effective cyclotron mass remains anged when the frequency is doubled or quadrupled).

3. The fact that no such singularities are present in the de Haas—van Alphen oscillations /s us to connect the appearance of the combination frequencies and the frequency doubling the term $\Delta\sigma_2$ in (1), which has no analogy in the expression for $\partial M/\partial H$. The contribution $\Delta\sigma_2$, which increases with increasing field H, diverges logarithmically if the width Γ of the lau level tends to zero. In contrast, $\Delta\sigma_1$ has a finite width as $\Gamma\to 0$ (this result is a consequence e integration, with respect to energy, of the singularities $|\phi|=3.7°$ and $n_1(\epsilon)\sim 1/\sqrt{\Gamma}$ and $\sim(1/\sqrt{\Gamma})^2$ for $\Delta\sigma_1$ and $\Delta\sigma_2$, respectively).

According to (2), the contribution from $\Delta\sigma_2$ predominates in the oscillations of $\partial\rho/\partial H$ and H, if the amplitude of the field-dependent part $n_0(\epsilon)$ of the state density is much larger than ionotonic component $n_0(\epsilon)$. In this case the amplitude of the second harmonic should increase ly

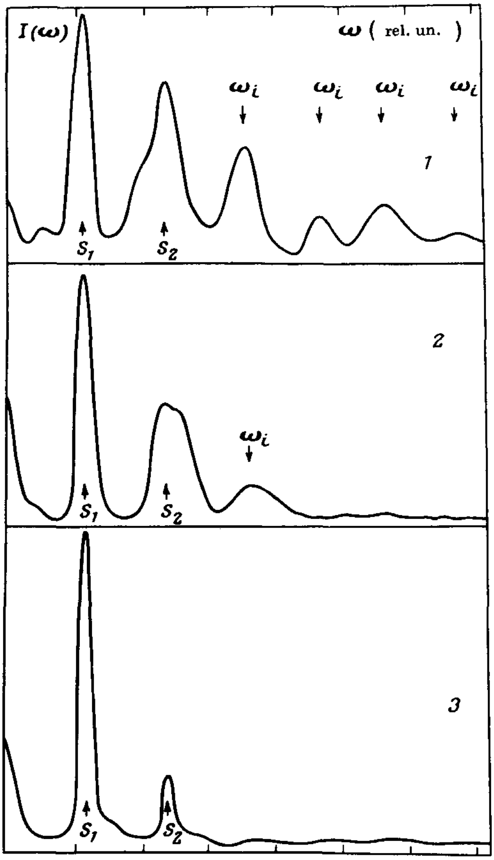


FIG. 2. Spectral composition $I(\omega)$ of the oscillations curves shown in Fig. 1: 1—$\partial\rho/\partial H$; 2—$\partial Z/\partial H$; 3—$\partial M/\partial H$.

$$\Delta\sigma_2 \sim n_1^2(\epsilon) \sim (\cos\omega t)^2 \sim \cos 2\omega t, \; t = 1/H.$$

An additional mechanism that increases the amplitude of the oscillating part $n_1(\epsilon)$ of the s density is possible in doped samples. At low temperatures, in doped samples, the princ scattering mechanism that determines the carrier relaxation time τ, and hence the width $\Gamma \sim 1/$ the Landau level, is scattering by ionized impurities. The effectiveness of this scattering is de mined by the screening radius r_0 of the impurity centers:

$$r_0^2 = (4\pi e^2 (n_0(\epsilon) + n_1(\epsilon)))^{-1},$$

where $n_0(\epsilon)+n_1(\epsilon)$ is the total density of states on the Fermi level. According to (5), the oscillat of the state density $n_1(\epsilon)$ give rise to oscillations of r_0 at the same frequency. The oscillations lead to oscillations of τ and Γ. When the Landau level coincides with ϵ_F, the component n increases sharply, r_0 decreases in accordance with (5), τ increases, and the width Γ of the le decreases, leading to a still greater increase of $n_1(\epsilon)$, etc. At $T=0$ in the absence of ca mechanisms other than scattering by ionized impurities, and at not too large quantum number the decrease of r_0 [meaning the increase of $n_1(\epsilon)$] will have, in our opinion, a resonant chara

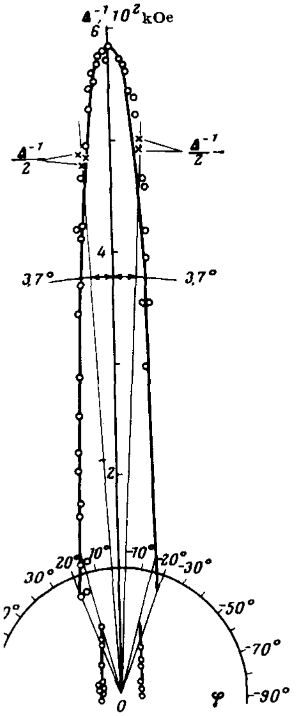


FIG. 3. Angular dependence of the frequency of the Shubnikov—de Haas oscillations for the sections of the electron ellipsoid of tellurium-doped Bi-Sb alloy when the field is oriented in the basal plane.

In the presence of two extremal sections S_1 and S_2, the oscillating part $n_1(\epsilon)$ of the state density the relaxation time τ will vary with frequencies ω_1 and ω_2 corresponding to the cross section S_1 S_2. In this case τ and $n_1^2(\epsilon)$ can be expanded in terms of the frequencies ω_1 and ω_2 and their nonics

$$\tau = \sum_j a_j \cos j\omega_1 + \sum_i a_i \cos i\omega_2; \quad n_1^2(\epsilon) = \sum_k b_k \cos k\omega_1 + \sum_l b_l \cos l\omega_2 . \qquad (6)$$

efore $\Delta\sigma_2 \sim \tau n_1^2(\epsilon)$ contains combination frequencies of the type $n\omega_1 + m\omega_2$ (n and m are gers).

In the de Haas—van Alphen oscillations, which are described by an expression analogous to term $\Delta\sigma_1$ in (2), there are no combination frequencies because of the weaker dependence of on the width Γ of the Landau level. In weak fields (Fig. 1) there are likewise no combination uencies in the spectra of the oscillations of $\partial\rho/\partial H$, $\partial Z/\partial H$, and $\partial M/\partial H$, in as much as the ratio $)/n_0(\epsilon)$ is small and according to (5) the oscillations of the screening radius r_0 are small.

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