ispersion approach to the calculation of three-article bound states and contributions of three-article unitarity

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The following are demonstrated: 1) the equivalence of the single-channel N/D method to the pole approximation in the Amado model, and 2) the need for taking into account three-particle, s-channel unitarity in the calculation (within the framework of the dispersion method) of the ground and excited states of a three-particle system.

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A dispersion method based on the use of the analytic properties of the scattering amplitude s been widely discussed in the theory of systems with few nucleons.[1–5] Recent investigations ve demonstrated the possibility of using this approach for systems containing antibaryons.[6] owever, in practical solutions of the N/D equations, it is customary to use a number of approxi-ations whose accuracy remains uncertain to this day. In particular, the following questions arise: what is the practical conversion of the method, i.e., how many Born diagrams it is necessary to e to reconstruct the total amplitude with the aid of the N/D equations? 2) What role is played in s case by the multiparticle unitarity condition? In the present article we answer both questions comparing the predictions of the dispersion method for bound states in a system of three rticles with the results of an exactly solvable three-particle model.

Consider a system of three identical particles with mass m, which interact via a separable tential with a form factor $\Gamma(p)=(p^2+\beta^2)^{-1}$ (Amado model).[7] This system is characterized by o parameters: the binding energy $\epsilon=\alpha^2/m$ of the two-particle subsystem and the interaction lius $r_0=1/\beta$. It is known that if the pairing potential is strong enough to bind each pair of rticles, then the system of three identical bosons has two bound states with $J^P=0^+$, namely a ep ground state (E_1) and a weakly bound excited state (E_2). Table I gives the value of E_1 and E_2 cases when ϵ and β correspond to triplet NN interactions[7] and to low-energy α-α scattering.[8] e consider below these two cases, in the first of which $\alpha r_0 \ll 1 (\zeta=\beta/\alpha=6.26)$, and in the second $\sim (\zeta=1.45)$, so that the relative positions of the dynamics singularities of the three-particle plitudes are substantially different.

TABLE I. Comparison of results obtained by solving the N/D equations with two-particle unita with the results of the exact solution of the Amado model. The bound-state energies (in MeV) reckoned from the threshold of the three-particle breakup.

	E_b	N/D equations		Amado model	
		$B^{(1)}$	$B^{(1)} + B^{(2)}_{\text{bare}}$	Pole approximation	Exact solution
$3N$ system	E_1	2.7	4.3	4.6	25.5
$\zeta = 6.26$	E_2	–	–	–	2.38
3α system	E_1	3.3	4.4	5.9	13.0
$\zeta = 1.45$	E_2	–	–	–	3.07

We consider the s-wave amplitude of the scattering of a particle by a bound state of two o particles $f(E)$, where E is the kinetic energy in the c.m.s.[3] Neglecting the contribution of the th particle unitarity and linearizing the two-particle unitarity condition with the aid of the subs tion $\tilde{f}=\tilde{n}/D$, we obtain for $D(x)$ the integral equation:

$$D(x) = 1 + \frac{2x}{\pi} \int_{\alpha} \frac{\tilde{\xi}(x')D(x')}{x + x'} dx',$$

where $\tilde{\xi}(x)$ is the dynamic jump of the Born amplitude; this amplitude is usually approximate the first few terms of the multiple-scattering series: $\tilde{f}=\tilde{B}^{(1)}+\tilde{B}^{(2)}+\cdots$.

The pole amplitude $\tilde{B}^{(1)}$ has two logarithmic cuts, which overlap at $\zeta<3$ and can parti cancel each other. The explicit expression for the jumps in our model is[4]:

$$\tilde{\xi}^{(1)}(x) = -\frac{4\pi\gamma}{\sqrt{3}} \left\{ \frac{1}{x^2}(\theta_1 - \theta_2) + (1 - \zeta^{-2})\left[\delta(x^2 - 3\zeta^2) - \delta\left(x^2 - \frac{\zeta^2}{3}\right)\right]\right\},$$

where

$$\gamma = \frac{\zeta(1+\zeta)^3}{(\zeta^2-1)^2}, \quad \theta_1 = \theta\left(x^2 - \frac{1}{3}\right)\theta(3 - x^2), \quad \theta_2 = \theta\left(x^2 - \frac{\zeta^2}{3}\right)\theta(3\zeta^2 - x^2).$$

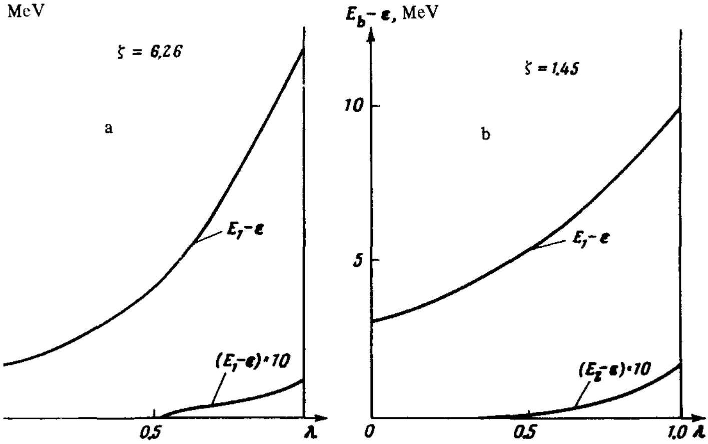
The triangular amplitude $\tilde{B}^{(2)}$ has an anomalous cut that begins at the point $x_a=\sqrt{3}$, a number of potential cuts that begin at the points x_p, which arranged at $\zeta=6.26$ in the follo sequence:

$$x_p = \frac{\sqrt{3}}{2}\zeta, \ \frac{\sqrt{3}}{5}(1+2\zeta), \ \frac{\sqrt{3}}{2}(1+\zeta), \ \frac{3\sqrt{3}}{5}\zeta, \ \frac{2\zeta+1}{\sqrt{3}}, \ \zeta\sqrt{3}.$$

The explicit form of the jumps of $\tilde{B}^{(2)}$ is quite complicated and will be presented elsehwere; hei confine ourselves to the limiting expression for $\tilde{\xi}^{(2)}$ at $\zeta \to \infty$:

$$\tilde{\xi}^{(2)}(x) = -\frac{64\pi}{3} \frac{1}{x^3} \ln \frac{3x - \sqrt{3}}{x + \sqrt{3}} \theta(x - \sqrt{3}).$$

The relative positions of the anomalous and potential cuts depend on the value o parameter ζ; in particular, at $\zeta<2$ the closest to the physical region is one of the potential



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As to the singularities of the more complicated diagrams, firstly, in our case of particles with l masses diagrams with two and more rescatterings have no anomalous singularities[10] that nd only on ϵ. Thus, as $r_0 \to 0$ the singularities of the pole and triangular diagrams account for ıe left-hand singularities of the amplitude. On the other hand, the potential singularities can r go off to infinity or have condensation points[4] with increasing number of rescatterings.

We can now ascertain the contribution of the higher Born diagrams by comparing the ctions of the N/D equations in the form (1) with allowance for the left-hand singularities of the amplitudes $\tilde{B}^{(1)}$ and $\tilde{B}^{(2)}$, on the one hand, with the exact solution of the Amado model in ·ole approximation, on the other. In this approximation the amplitude f has (in accordance the terminology of[3]) a two-particle unitary jump Δ_5 and a "crossing" three-particle unitary Δ_{4n}, but does not contain a "direct" three-particle cut. The results are given in Table I. From olution of the N/D equations with a triangular diagram containing a bare propagator, we ned a binding energy 4.3 MeV, while the exact answer is 4.6 MeV; thus, the contribution of ıiscarded diagrams is $\sim 10\%$. This result is very important from the point of view of the rsion method as a whole. It shows that at $\alpha r_0 \ll 1$ the pole and triangle diagrams account with accuracy for the entire dynamic input amplitude. In the region $\alpha r_0 \sim 1$, the influence of the rded terms is more significant and amounts to $\sim 25\%$, but here, too, one can speak of nable qualitative agreement with the exact solution.

We examine now the contribution of the three-particle unitarity. We note first that in the pole ›ximation the Amado model yields one shallow level, while the exact solution contains two ;. The question is therefore whether the shallow level corresponds to an excited state of the -particle system or is a very weakly bound ground state. This question can be answered by ducing the parameter $\lambda (0 \leqslant \lambda \leqslant 1)$, which determines the contribution of the direct three-:le intermediate states, and by tracing the trajectory of the shallow level with changing λ. This ıveniently done technically by replacing

$$\tau(y) = \left\{ 1 - \frac{(1+\zeta)^2}{(\zeta + \sqrt{1-y})^2} \right\}^{-1} \qquad (4)$$

in the Faddeev equation by

$$r(y,\lambda) = \frac{1+\zeta}{y} + \lambda\left\{1 + \frac{\zeta(1+\zeta)}{1+\sqrt{1-y}}\,\frac{1}{1+2\zeta+\sqrt{1-y}}\right\}$$

and considering the behavior of the Faddeev solutions as a function of λ. Our calculations sh that with changing λ the shallow levels deepens continuously ($dE_1/d\lambda < 0$), and becomes ground level at $\lambda = 1$, while the excited level first manifests itself at a certain $\lambda > 0$. Fig. 1 shows behaviors of $J^P = 0^+$ levels in $3N$ and 3α systems as a function of λ. The excited level first appe at $\lambda = 0.6$ in the case of the $3N$ system and $\lambda = 0.4$ in the case of 3α system. Thus, in the calculat of the bound states of three-particle systems, allowance for multiparticle unitary is essential.5) T applies equally well to four-particle bound states.[11] We indicate also that multiparticle unitarit essential for the explanation (within the framework of the N/D method) of the two princi qualitative effects in systems with few nucleons. It is well known that the kernel of the Fadd equation diverges in the two limiting cases $r_0 \to 0$ and $\alpha \to 0$, which leads to the Thomas theor and to the Efimov effect.[12] Yet it is easily seen that the N/D equations with two-particle unita remain of the Fredholm type in these limiting cases. Therefore the divergence of the solutions the N/D equations as $r_0 \to 0$ and $\alpha \to 0$ can be due only to inclusion of three-particle unitarity

We note in conclusion that physical situations are possible in which the contribution of three-particle unitarity is inessential. An example of such a situation is the quartet n-d scatteri when by virtue of the Pauli principle all three particles cannot be simultaneously at short distan from one another. In this case even the simplest approximation in the single-channel N/D metl yields satisfactory results.[1]

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3)Following[1], we use henceforth the dimensionless amplitudes $\bar{f}(y) = 2\alpha f(y)/\sqrt{3}$ a $\bar{B}(y) = 2\alpha B(y)/\sqrt{3}$ and the dimensionless variable $y = -x^2 = E/\epsilon$; the first sheet of the ene plane corresponds to values Re $x \geqslant 0$.
4)We note that $\xi^{(1)}$ has δ-like singularities that cause the function $n(x)$ to have poles at $x = \pm\zeta/$ and $\zeta\sqrt{3}$. The poles on the second sheet of the energy plane, $x = -\zeta/\sqrt{3}$ and $-\zeta\sqrt{3}$, cancelled by the corresponding poles of the function $D(x)$,[9] so that the total amplitude has p on the first sheet at $x = \zeta/\sqrt{3}$ and $\zeta\sqrt{3}$.
5)The phenomenological contribution of the three-particle intermediate states can be taken i account in single-channel N/D equations by introducing CDD poles.[5]

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