

Production of four pions in e^+e^- collisions and partial conservation of axial current

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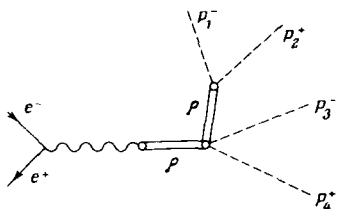
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It is shown that allowance for partial conservation of the axial current (PCAC) and for the identity of the final pions in the reaction $e^+e^- \rightarrow \rho\pi^+\pi^-$ leads to a total cross section energy dependence that agrees with experiment. The ratio of the cross sections of the reactions $e^+e^- \rightarrow \rho\pi^+\pi^-$ and $e^+e^- \rightarrow \rho\pi^0\pi^0$ is calculated.

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The cross section of the reaction $e^+e^- \rightarrow 2\pi^+2\pi^-$ was recently measured in the Orsay and Novosibirsk colliding beam installations at energies \sqrt{s} ranging from 915 to 1340 MeV.^[1,2] The measured cross section and its energy dependence differ noticeably from the theoretical predictions.^[3-6] In the present paper we consider the reaction $e^+e^- \rightarrow \rho \rightarrow \rho\pi^+\pi^- \rightarrow 2\pi^+2\pi^-$ with allowance for



G. 1.

identity of the final pions, in contrast to the quasi-two-particle or quasi-three-particle approach in^[3-6] and show that the energy dependence of the total cross section in the near-threshold region turns out to be less steep.

We write down the amplitude of the transitions $\rho \rightarrow \rho\pi\pi$, with account taken of the Adler rule soft pions, in the form (this question will be discussed in greater detail in our paper^[7] with the analogous process $\psi' \rightarrow \psi\pi\pi$ as an example):

$$M = g_{\rho\rho\pi\pi} [(\epsilon_1 \epsilon_2)(k_1 k_2) - (\epsilon_2 k_1)(\epsilon_1 k_2)] (p_1 p_2), \quad (1)$$

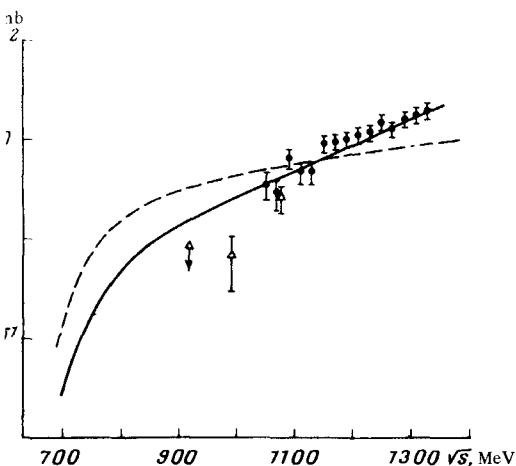
where $\epsilon_{1,2}$ are the 4-polarizations, $k_{1,2}$ are the 4-momenta of the ρ mesons, and $p_{1,2}$ are the 4-momenta of the pions. Allowance for the permutations of the identical pions on the diagram of Fig. yields for the square of the matrix elements, averaged over the polarizations of the initial leptons

$$|M|^2 = 2g_{\rho\rho\pi\pi}^2 \sum_{i,k=1}^4 T_i T_k^* [Q^2 (R_i R_k) + (q R_i)(q R_k)], \quad (2)$$

where

$$T_i = (p_3 p_4) / [m_\rho^2 - (p_1 + p_2)^2 - im_\rho \Gamma_\rho],$$

$$R_{1\mu} = (Q p_2)_\mu p_{1\mu} - (Q p_1)_\mu p_{2\mu}, \quad Q = p_+ + p_-, \quad q = p_+ - p_-;$$



G. 2. Total cross section of the reaction $e^+e^- \rightarrow \rho\pi^+\pi^-$ with (solid curve) and without (dashed) allowance for PCAC. Points with error bars—experimental data from Orsay (triangles) and Vosibirsk (circles).

TABLE I.

\sqrt{s} , GeV	0.6	0.8	1.0	1.2	1.4	1.6	5.0
$\sigma_{\rho\pi^+\pi^-}/\sigma_{\rho\pi^0\pi^0}$	4.0	3.9	3.7	3.2	2.9	2.7	2.4

T_i and $R_{i\mu}$ at $i=2,3,4$ are obtained from T_1 and $R_{1\mu}$ by permutations of the 4-momenta of t identical pions, $p_{1,3}$ are the 4-momenta of π^- , $p_{2,4}$ are the 4-momenta of π^+ , p_{\pm} are the 4-momenta of e^{\pm} , and m_{ρ} and Γ_{ρ} are the mass and width of the ρ meson. We integrate over the phase value of the 4 pions by the Monte Carlo method, using the random-star algorithm proposed in^[8]. The fit to the common unknown constant $g_{\rho\rho\pi\pi}$ yields the total reaction cross sections represented in Fig. 1 by the solid curves together with the experimental data.^[1,2] The dashed curve shows the cross section calculated using a matrix element without allowance for Adler's rule. It is seen that the solid curve describes the experimental data much better, i.e., allowance for the PCAC is quite essential.

When the cross section of the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ is determined from experiment it is important to know the contribution of the reaction $e^+e^- \rightarrow \rho\pi^0\pi^0$. If the two pions are in a state with isospin 0, then isotopic invariance predicts a value 2 for the cross section ratio $\sigma_{\rho\pi^+\pi^-}/\sigma_{\rho\pi^0\pi^0}$. Allowance for the identity of the final pions changes this ratio, since the process $e^+e^- \rightarrow \rho\pi^0\pi^0$ is described by only one diagram of Fig. 1, and in the case $e^+e^- \rightarrow \rho\pi^+\pi^-$ interference between the final states sets in because of the large width of the ρ meson. Table I illustrates the dependence of the cross-section ratio on the energies. It is seen that near the threshold there is a strong interference that leads to a cross-section ratio of 4, which is the maximum possible in accord with isotopic invariance^[9] (this corresponds to the production of two pairs of pions in an S wave with isospin 2) the role of the interference decreases with energy, and the cross-section ratio tends to 2 only at very high energy, when the ρ meson is produced with a large momentum.

Thus, allowance for PCAC, and also the identity of the final pions in the reaction $e^+e^- \rightarrow \rho\pi^+\pi^-$ leads to a total-cross-section energy dependence that agrees with experiment, and alters noticeably the cross section ratio $\sigma_{\rho\pi^+\pi^-}/\sigma_{\rho\pi^0\pi^0}$. It would be of great interest to check this model against more accurate measurements of the cross section at energies \sqrt{s} from 800 to 1000 MeV.

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