elaxation of pion fields in nuclear matter

V. M. Galitskiĭ and I. N. Mishustin

I. V. Kurchatov Institute of Atomic Energy
(Submitted August 18, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **26**, No. 7, 566–569 (5 October 1977)

We consider the behavior of a classical pion field in nuclear matter in the case when its initial value is not equal to the equilibrium value. Assuming that the principal dissipation mechanism is Landau damping, we obtain the time of relaxation of the pion field to the equilibrium value, which turns out to be much shorter than the collision time of heavy ions with energies of several hundred MeV per nucleon.

PACS numbers: 21.65.+f

The role of meson degrees of freedom in atomic nuclei have been extensively investigated ently. Starting with Migdal's work in 1971,[1] the theoreticians and experimenters have been rticularly interested in the problem of pion condensation in nuclear matter and in the possible istence of anomalous—superdense—nuclei. In connection with development of experiments with gh-energy heavy ions, aimed at checking on these theoretical predictions, great importance aches to the investigation of the dynamic characteristics of the pion condensate as a function of e degree of compression of the nuclear matter.

It follows from the theory of pion condensation[2] that near the phase-transition point the issical pion field $\phi(\mathbf{r},t)$ is described by the equation (for the sake of argument we shall take $\phi(\mathbf{r},t)$ mean the field of charged pions):

$$\{[\alpha\hat{\omega}^2 + 2i\beta\hat{\omega} - \omega_0^2 - \gamma(\hat{\mathbf{k}}^2 - k_0^2)^2] - \lambda|\phi(\mathbf{r},t)|^2\}\phi(\mathbf{r},t) = 0. \tag{1}$$

ere $\hat{\omega}$ and $\hat{\mathbf{k}}$ are the operators of differentiation with respect to time and coordinates: $\hat{\omega}=i\partial/\partial t$, $= -i\partial/\partial\mathbf{r}$. The expression in the square brackets constitute in the momentum representation the st terms of the expansion of the reciprocal pion propagator in ω and k near the values $\omega=0$ and $=k_0$ corresponding to the equilibrium pion field. The quantity ω_0^2 has a behavior typical of a cond-order phase transition: $\omega_0^2=\eta(n_c-n)$, i.e., it reverses sign at a critical nucleon density $n=n_c$ he parameters α, β, γ, and η are expressed in terms of the polarization operator of the pions in e nuclear matter $\Pi(k,\omega)$ in the momentum representation:

$$\alpha = 1 - \frac{\partial}{\partial\omega^2}\,\mathrm{Re}\,\Pi; \quad \beta = \frac{1}{2}\,\frac{\partial}{\partial\omega}\,\mathrm{Im}\,\Pi; \quad \gamma = \frac{1}{2}\,\frac{\partial^2\Pi}{\partial(k^2)^2}; \quad \eta = -\frac{\partial\Pi}{\partial n}. \tag{2}$$

ll the derivatives in (2) are taken at $\omega=0$, $k=k_0$, $n=n_c$. The values of k_0 and n_c are obtained from e dispersion equations for the pions at $\omega=0$. An explicit expression for Π, calculated in a fficiently realistic model, can be found in[2]. Here we present an explicit expression for β at $<2p_F$:

$$\beta = \frac{f^2 k_0}{2\pi}\left(\frac{m^*}{m_\pi}\right)^2(1+g^-)^{-2}, \tag{3}$$

ere $f=1.0$ is the coupling constant of the N interaction, $m^*\approx 0.9m_N$ is the effective mass of the icleons in the nuclear matter, m_π and m_N are the masses of the pions and nucleons in vacuum, d g^- is the constant of the local spin-spin interaction of the nucleon quasiparticles.[2,3]

At a value $g' = 1.6$, which leads to n_c close to the nuclear density, the parameters of (1) take c the values (in pion units: $\hbar = c = m_h = 1$):

$$a = 5, \quad \beta = 2, \quad \gamma = 0.25, \quad \eta = 6, \quad k_o = 2.4. \qquad ($$

The quantity λ in (1) characterizes the effective interaction of the pion quasiparticles i nuclear matter. Its determination entails great computational difficulties.[4] Rough estimates yiel $\lambda \sim 10$.

From the point of view of the relaxation properties of the system, particular importan attaches in Eq. (1) to the second term in the square brackets, which is connected with tl imaginary part of the polarization operator by relations (2) and (3). It is known that th imaginary part is due mainly to the decay of the collective excitations into a particle and hole fro the continuous spectrum (Landau damping). At $n > n_c$, the initial state with $\phi = 0$ is an excite state of the system and it is precisely the Landau damping which provides an effective mechanis for diverting the excess energy of the pion field to the nucleon degrees of freedom.

We choose the coordinate dependence of the pion field in the simplest form compatible wit Eq. (1), namely $\phi(\mathbf{r},t) = \phi(t)\exp(ik_0 z)$, and trace its evolution in time at $n > n_c$. To obtain tl qualitative picture, we linearize Eq. (1) by expanding $\phi(t)$ near its equilibrium valu $|\phi_{eq}| = (-\omega_0^2/\lambda)^{1/2}$. Introducing the notation $\chi(t) = \phi_{eq} - \phi(t)$, we obtain a damped solution in tl form:

$$\chi(t) = \chi_o \left[\frac{\mathrm{sh}\left(\frac{\beta}{a}\sqrt{1-\xi^2}\,t\right)}{\sqrt{1-\xi^2}} + \mathrm{ch}\left(\frac{\beta}{a}\sqrt{1-\xi^2}\,t\right) \right] \exp\left(-\frac{\beta}{a}t\right), \qquad ($$

$$\xi^2 \equiv -\frac{2a}{\beta^2}\omega_o^2 \geqslant 0.$$

Here χ_0 is the initial deviation, and the initial value of $\dot{\chi}$ is assumed to be zero. It follows therefo that at $\xi^2 < 1$ the field reaches it equilibrium value in an aperiodic regime, and at $\xi^2 > 1$ dampe oscillations take place. The case $\xi^2 = 1$ corresponds to critical damping. The relaxation time behaves near the critical point ($\omega_0^2 \to 0$) like $-\beta/\omega_0^2$, i.e., it has a singularity at $n = n_c$. Wi increasing distance from the critical point, τ decreases quite rapidly and at $\xi^2 \gg 1$, which corr sponds to $(n - n_c)/n_c \gtrsim 0.2$, it approaches the constant value

$$\tau_o = \frac{a}{\beta} \approx 2.5 \frac{\hbar}{m_\pi c^2} = 1.2 \cdot 10^{-23}\ \mathrm{sec}. \qquad ($$

We note that in the case $|a\omega_0^2/\beta^2| \ll 1$, which is realized near the critical point, we can negle the first term in (1), and this equation reduces then to a Bernoulli equation and admits of an exa analytic solution:

$$|\phi(t)|^2 = |\phi_{eq}|^2 \left[1 + \left(\frac{|\phi_{eq}|^2}{|\phi_o|^2} - 1 \right) \exp\left(\frac{\omega_o^2}{\beta} t \right) \right]^{-1}, \quad \omega_o^2 < 0.$$

Here $|\phi_0|^2$ is the initial value of the field, which is determined by quantum or thermal fluctuatior The characteristic time of variation of the field in this case, $-\beta/\omega_0^2$, coincides with that obtain from (5) as $\xi^2 \to 0$, thus justifying the qualitative calculations given above.

The collision of two heavy nuclei is characterized in nonrelativistic kinematics by a characte istic time

$$\tau_{coll} = 4 r_0 A^{1/3}\left(\frac{2w}{m_N}\right)^{-1/2}$$

ıere $r_0 = 1.2 \times 10^{-13}$ cm and w is the energy per nucleon of the incident ion in the lab. For ample, at $w = 250$ MeV/A and $A^{1/3} = 6$ we obtain $\tau_{coll} = 1.4 \times 10^{-22}$ sec. Comparison with (6) ows that $\tau_0 \ll \tau_{coll}$.

Thus, with the exception of the immediate vicinity of the critical point, the relaxation time of e pion field turns out to be much shorter than the lifetime of the cluster of compressed nuclear ıatter produced as a result of the collision. This allows us to regard the process of collision of avy nuclei with energy of several hundred MeV per nucleon as adiabatic for the pion degrees of :edom and to use for the description of the collision dynamics the parameters of the equilibrium ɔn condensate, as was done in[4]. This, however, raises the question of the additional system ating via the relaxation mechanism described above. The thermal properties of the pion conden- te will be considered in a subsequent paper.

The authors are grateful to V.A. Khodel' for useful discussions.

..B. Migdal, Zh. Eksp. Teor. Fiz. **61**, 2209 (1971) [Sov. Phys. JETP **34**, 1184 (1971)].
..B. Migdal, O.A. Markin, and I.N. Mishustin, Zh. Eksp. Teor. Fiz. **66**, 443 (1974) [Sov. Phys. ETP **39**, 212 (1974)].
..B. Migdal, Teoriya knechnykh fermi-sistem i svoĭstva antomnykh yader (Theory of Finite Fermi ystems and of the Properties of Atomic Nuclei), Nauka, 1965.
..B. Migdal, O.A. Markin, and I.N. Mishustin, Zh. Eksp. Teor. Fiz. **70**, 1592 (1976) [Sov. Phys. ETP **43**, 830 (1976)].
.M. Galitskiĭ and I.N. Mishustin, Preprint IAE-2873, Moscow, 1977.