Gauge invariance in supergravitation

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Starting from the gauge-invariance requirement, new Feynman rules that include four-fermion interaction of ficticious particles are constructed in the theory of supergravitation.

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The recently proposed supergravitation theory[1,2]—a theory in which the gravitons an particles with spins 3/2 are connected by local supersymmetry transformations—have much bett renormalizability properties[3] than ordinary gravitation theory. There is hope that a gauge supe symmetry will make it possible to construct a renormalized gravitation theory.

The construction of a quantum theory of gravitation is faced, however, with a number unsolved problems, one of which is connected with the fact that the transformations of the loc supersymmetry do not correspond to a group with closed algebra,[2] as was the case with all tl heretofore studied gauge theories—in the Yang-Mills theory and in the ordinary gravitatic theory. In supergravitation, the gauge invariance of the theory, which is needed to prove tl renormalizability and unitarity, cannot be proved in the usual manner for lack of a closed algebr

The simplest method of investigating gauge invariance of quantum theory is connected wi the transformations of Becchi-Rouet-Stora-Tyutin (BRST),[4] who noted that although the inva iance of the classical action is lost when a gauge condition and the action of a fictitious particle added in quantum theory, the resultant effective action has a new nonlinear global invariance wi an anticommuting parameter between the gauge field and the fictitious particles.

We consider an arbitrary gauge theory, in which the classical action $S_{cl}[\Phi]$ is invariant certain transformations of the gauge field Φ^i (in supergravitation, $\Phi^i=\{e^a_\mu,\Psi^\alpha_\mu\}$)

$$\Phi^i \to \Phi^i + R^i_a[\Phi]\,\xi^a(x), \qquad ($$

$$\frac{\delta S}{\delta \Phi^i} \equiv S_{,i} \quad ; \quad S_{,i}\, R^i_a = 0, \qquad ($$

$\xi^a(x)$ is an arbitrary infinitesimally small transformation parameter. The effective quantum-fi action is then constructed in accordance with the rule[5]

$$S_{\text{eff}} = S_{\text{cl}} + S_{\text{gauge}} + S_{\text{f.p.}},$$

where the term that fixes the gauge condition is of the form

$$S_{\text{gauge}} = 1/2\,\Phi^i F_{ja}\,\tilde{\gamma}^{a\gamma} F_{i\gamma}\Phi^i,$$

and the action of the fictitious particles c^β is of the form

$$S_{\text{f.p.}} = \bar{c}^{\gamma} F_{i\gamma} R^i_\beta\, c^\beta .$$

Here S_{eff} is invariant to the BRST transformations[4] with a unity Jacobian

$$\Phi^i \to \Phi^i + R^i_a c^a \Lambda,$$

$$c^{\beta} \to c^{\beta} - 1/2 f^{\beta}_{\alpha\gamma} c^{\alpha} c^{\gamma} \Lambda, \tag{7}$$

$$\bar{c}^{\gamma} \to \bar{c}^{\gamma} - \Phi^{j} F_{j\alpha} \tilde{\gamma}^{\alpha\gamma} \Lambda, \tag{8}$$

ıere Λ is the anticommuting parameter, under the condition that the transformations (1) form a)sed algebra with structure coefficients

$$f^{\beta}_{\alpha\gamma}, \text{ and } \frac{\partial \tilde{R}^{i}_{\alpha}}{\partial \Phi^{i}} \equiv R^{i}_{\alpha, i} = 0 ; \qquad f^{\beta}_{\alpha\beta} = 0. \tag{9}$$

however, just as in the case of supergravitation, the gauge transformations form a closed ıduated algebra only on the equations of motion, i.e., if the exchange Lie bracket is of the form[2]

$$[R^{i}_{\alpha, j} R^{j}_{\beta} \} = R^{i}_{\gamma} f^{\gamma}_{\alpha\beta} + \eta^{ij}_{\alpha\beta} S_{,j} \quad , \tag{10}$$

d S_{eff} is constructed in accordance with formulas (3–5), then S_{eff} is no longer invariant under the nsformations (6–8)[1]

$$\delta S_{\text{eff}} = 1/2 \bar{c}^{\gamma} F_{i\gamma} \eta^{ij}_{\alpha\beta} S_{,j} c^{\alpha} c^{\beta} \Lambda \equiv - S_{,j} \bar{\delta} \Phi^{j} \Lambda . \tag{11}$$

seen from (11), it is now natural to take a new transformation of the field Φ^{i} in the form $\delta\Phi^{i}$, ere

$$\tilde{\tilde{\delta}} \Phi = (R^{i}_{\alpha} c^{\alpha} + \bar{\delta} \Phi^{i}) \Lambda = \tilde{\tilde{R}}^{i}_{\alpha} c^{\alpha} \Lambda . \tag{12}$$

re S_{cl} is no longer invariant to the transformations (12) and its variation cancels out (11) exactly. e new action for the fictitious particle will be determined from the requirement that, as in the ıal case, the variation of the gauge condition under the transformation (12) be cancelled out by transformation (8) for the fictitious antiparticle $\bar{c}\gamma$. We then obtain

$$\tilde{\tilde{S}}_{\text{f.p.}} = \bar{c}^{\gamma} F_{i\gamma} R^{i}_{\beta} c^{\beta} - 1/4 \bar{c}^{\gamma} c^{\beta} F_{i\gamma} \eta^{ij}_{\alpha\beta} F_{j\delta} c^{\alpha} \bar{c}^{\delta} \equiv \bar{c}^{\gamma} F_{i\gamma} \tilde{\delta} \Phi^{i} \tag{13}$$

$$\tilde{\delta} \Phi^{i} \equiv R^{i}_{\alpha} c^{\alpha} + 1/2 \bar{\delta} \Phi^{i}. \tag{14}$$

now find the variation of (13) when the gauge field (12) is changed. It is equal to

$$\delta \tilde{\tilde{S}}_{\text{f.p.}} = \bar{c}^{\gamma} F_{i\gamma} \tilde{\tilde{\delta}} (\tilde{\delta} \Phi^{i}) = (- S_{,i} \bar{\delta} \Phi^{i} + 1/2 \bar{c}^{\gamma} F_{i\gamma} \tilde{\tilde{R}}^{i}_{\xi} f^{\xi}_{\alpha\beta} c^{\alpha} c^{\beta}) \Lambda . \tag{15}$$

the derivation of (15) we used the following facts: 1) from the Jacobi identity for the cyclic mutations of three successive transformations (1) with allowace for (10) it follows that in ergravitation (with symmetrization with respect to $\alpha\beta\gamma$)

$$- \eta^{ij}_{\xi\gamma} f^{\xi}_{\alpha\beta} + \eta^{ij}_{\alpha\beta,\kappa} R^{\kappa}_{\gamma} + R^{i}_{\gamma,\kappa} \eta^{\kappa j}_{\alpha\beta} + \eta^{i\kappa}_{\alpha\beta} R^{i}_{\gamma,\kappa} = \tilde{\tilde{\eta}}^{ij\kappa}_{\alpha\beta\gamma} S_{,\kappa}, \tag{16}$$

n supergravitation the function η is equal to zero, 3) in supergravitation the following relation ds

$$\eta^{ij}_{\alpha\beta,\kappa} \eta^{\kappa l}_{\gamma\delta} = 0, \tag{17}$$

n the left-side of (15), $R^{i}_{\alpha,j} \eta^{jl}_{\gamma\xi}$ can be replaced, by virtue of the symmetry of $\bar{c}\gamma F_{i\gamma}$ and $\bar{c}^{\delta} F_{j\delta}$, by $R^{l}_{\alpha,j}$.

Equation (15) means that the variation S_{cl} cancels out the first term, and the variation of the itious particle c^{β} (7) cancels the second term in δS_{fp}.[2] We have thus shown that S_{eff} [Eqs. (3), (13)] is invariant to the transformations (12), (7), and (8).[3] In the generating functional W of theory

$$W = \int d\Phi\, d\bar{c}\, dc \exp i \{ S_{\text{eff}} + J_i \Phi^i + \bar{K}_a c^a + \bar{c}^\gamma K_\gamma \}$$ (

the variation applies then only to terms with forces

$$\delta W = i < J_i \tilde{\tilde{\delta}} \Phi^i + \bar{K}_a \delta c^a + \delta \bar{c}^\gamma K_\gamma > .$$ (1

Differentiating (19) with respect to J_i, with respect to k^γ, and with respect to Λ, we obtain t Slavnov-Taylor identity[7] in the form

$$i < F_{ja} \Phi^j \tilde{\gamma}^{a\gamma} \Phi^i + \bar{c}^\gamma \tilde{\tilde{R}}^i_a c^a |_{J=k=k=0} = 0 .$$ (2

It is easy to verify that the left-hand side of (20) is equal to $\delta W / \delta F_{i\gamma}$, and by virtue of (20) t generating functional of the theory on the mass shell is independent of the choice of the gau conditions $F_{i\gamma}$, Q.E.

We have learned after deriving the principal results of the paper, formulas (13) analogous the Feynman rule in supergravitation were obtained by Vasiliev and Fradkin[8] by the canoni quantization method.

1)Variation of S_{eff} in a certain concrete gauge was first calculated in[6].

2)The Jacobian of the tansformation is equal to 1 under condition (9) and $\eta^{ij}_{\alpha\beta,i} = 0$.

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