Determination of the $\pi\pi$ scattering length in the effective-radius approximation

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We present the results of the calculation of S and P scattering wavelengths in the effective-radius approximation, on the basis of a phase shift analysis of $\pi^-\pi^+$ interaction in the interval $360 < m_{\pi\pi} < 960$ MeV.

PACS numbers: 13.75.Lb

Interest in the study of the $\pi\pi$-interaction constants, including the determination of the scattering lengths, has increased in recent years. This is caused both by the very great ambiguity of the available experimental data and by the importance of the knowledge of the pion—pion scattering lengths for the choice of the true effective Lagrangian of the pion—pion interaction.

Various methods can be used to determine scattering lengths from experimental data: from data on K decay; from the cross sections of the reactions $\pi N \rightarrow \pi\pi N$ near the threshold; with the aid of the Roĭ equations from the phase curves, etc. All these methods have their advantages and shortcomings. We used a "purely experimental" method for obtaining the $\pi\pi$-scattering lengths, namely, we used the effective-radius approximation to determine the maximum $\pi\pi$-scattering amplitudes at threshold from the phase curves.
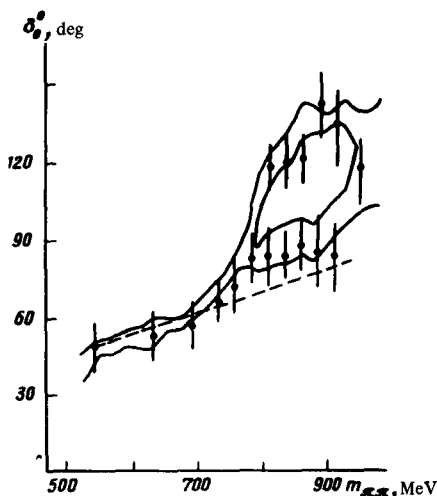
FIG. 1.

From the data on the reaction $\pi^- p \rightarrow \pi^- \pi^+ n$ at $p_{\pi^-} = 4.5$ GeV/c we determined the S and P wave phase shifts of the $\pi^- \pi^+$ scattering in the region $360 < m_{\pi\pi} < 960$ MeV. To find the phase shifts we used the cross sections and angular distributions extrapolated to the pion pole, as well as energy-dependent and energy-independent methods. The procedure used to obtain the phase shifts by the energy-independent method is described in detail in[1].

The phase shift δ_1^1, as expected, can be well fitted to a Breit–Wigner curve, and the values of the phase shifts δ_0^0 are shown in Fig. 1. The points and the dashed curve are the results of the energy-independent and energy-dependent analyses, respectively. For comparison, the possible range of values of δ_0^0 from[2] is shown.

To calculate the scattering lengths we used the effective-radius approximation:

$$k^{2l+1} \operatorname{ctg} \delta_l^T = \frac{1}{a} + \frac{1}{2} k^2 r_l^T . \tag{1}$$

Here k is the momentum of the pion in the dipion rest system, while l and T are the spin and isotopic indices, respectively.
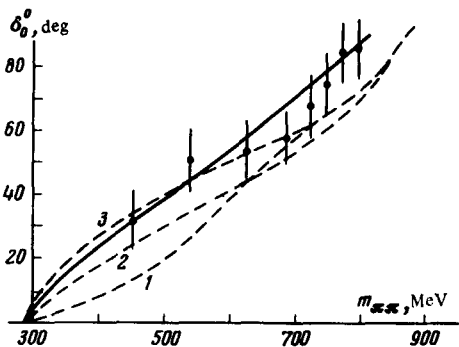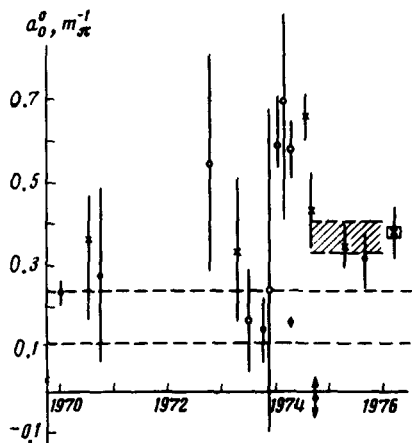

FIG. 2.

FIG. 3.

For the lengths a_1^1 we obtained from the energy-independent analysis the value $a_1^1 = (0.029 \pm 0.006)\mu^{-3}$. This agrees sufficiently well with the known experimental and theoretical values, which lie respectively in the ranges

$$0.027 < (a_1^1)_{\exp} < 0.050 \ \mu^{-3},$$

$$0.028 < (a_1^1)_{\text{theor}} < 0.035 \ \mu^{-3}.$$

From the values of the phase shift δ_0^0 in the region of the unique solution (up to 800 MeV), we obtained the following values:

$$a_o^{\ o} = (0.39 \pm 0.07)\mu^{-1}; \qquad r_o^{\ o} = (-0.94 \pm 0.13)f. \tag{2}$$

(The effective radius was also regarded as a free parameter.)

At a confidence level of 50%, the region of possible values of a_0^0 is

$$0.29 < a_o^o < 0.62 \ \mu^{-1}. \tag{3}$$

Thus, the result can be increased relatively easily, but can be decreased with difficulty. An approximation of the phase curve is shown in Fig. 2. The solid curve is the result of a calculation by formula (1), and the dashed curves are the results of[3] for the following: 1) $a_0^0 = -0.05 \ \mu^{-1}$; 2) $a_0^0 = 0.16 \ \mu^{-1}$; 3) $a_0^0 = 0.6$ μ^{-1}. It appears that the best agreement with our points would be given by the curve for $a_0^0 \sim 0.4 \ \mu^{-1}$.

To check on the validity of the use of formula (1) in a region that is relatively far from the threshold, calculations were performed with the first points of the phase curve. The results agree with one another and with the average results (2). That the procedure is a closed one was verified in addition by calculating the scattering lengths from the cross sections extrapolated to the $\pi^-\pi^+$ pole. The cross section was represented by a sum of partial cross sections, and the phase shifts were parametrized in the following manner:

$$\delta_o^o = \text{arc ctg}\left(\frac{1}{ka_o^o} + \frac{1}{2} \ k r_o^{\ o}\right),$$

$$\delta_1^1 = \text{arc tg}\left\{\frac{m_\rho \Gamma_\rho}{m_\rho^2 - m_{\pi\pi}^2}\left(\frac{k}{k_\rho}\right)^3\right\} \tag{4}$$

Taking into account the smallness and the weak influence of δ_0^2 on the other phases, the values of δ_0^2 were taken from other works.[4] The value obtained for a_0^0 agreed with that given above.

Thus, from the obtained phase shifts we get, in the effective-radius approximation, a stable value $a_0^0 \sim 0.4\ \mu^{-1}$. Figure 3 shows a comparison of our results with the data of others.[5] (The results are arranged in the figure from left to right in the same sequence as the references.)

The circles are from results on K decays, the points—from the reactions $\pi N \to \pi\pi N$ near the thresholds, the crosses—from the $\pi\pi$ phase shifts in the effective-radius approximation. The square with the cross in it is our present result. It is seen that although recently the experimental errors have become smaller, the picture still remains unclear. The experimental results obtained by various methods (and even within the limits of the same method) differ greatly from one another. There are discrepancies also in the theoretical predictions. Thus, the theoretical models such as that of Weinberg predict values of a_a^0 in the range $0.12 < a_0^0 < 0.25\ \mu^{-1}$ [6] (the region between the dashed lines in the figure), whereas in[7], where the Veneziano model was used, a value $a_0^0 \sim 0.4\ \mu^{-1}$ was obtained (shaded region).

We assume that the situation with $(a_0^0)_{exp}$ can be improved by studying the $\pi^0\pi^0$ state. It is known that both the effective-radius method and the dispersion approach with the aid of the Roĭ equations are based on the behavior of the δ_0^0 phase. It is difficult, however, to separate this phase from the $\pi^-\pi^+ \to \pi^-\pi^+$ channel, because of the dominant contribution of the P wave.

To study the $\pi^0\pi^0$ state we can use the reaction $\pi^-p \to \pi^0\pi^0n$. An investigation of this state will help determine the behavior of the δ_0^0 phase shift in the region ~ 800 MeV, and help solve the "up-down" problem above 1 GeV. These data, in turn, will permit a more accurate determination of the value of $(a_0^0)_{exp}$. We are presently planning an investigation of the reaction $\pi^-p \to \pi^0\pi^0n$ using a xenon bubble chamber with a pulsed magnetic field of 70 kG and a hydrogen target.

In conclusion, the authors are sincerely grateful to I. I. Gurevich for useful discussions and constant interest in the work.

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