Increase of spatial fluctuations in superconductors near the Anderson transition

L. N. Bulaevskiĭ and M. V. Sadovskiĭ
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

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The superconducting order parameter has been found to exhibit strong, impurity-induced spatial fluctuations over a certain temperature interval near T_c. Far from the Anderson localization threshold, this interval is very narrow in comparison with the interval of strong thermodynamic fluctuations, and the superconducting order parameter is a self-averaging quantity. Because of the disorder, the fluctuations near the localization threshold are appreciable over the entire region of manifestation of the superconductivity.

In the work of Bulaevskiĭ and Sadovskiĭ[1] and Kapitulnik and Kotliar[2] the theory formulated by Abrikosov and Gor'kov and by Anderson for dirty three-dimensional superconductors with $k_F l \gg 1$ was extended to compounds with a very high level of disorder which are near the Anderson metal-insulator transition (in these compounds

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the mean free path of an electron $l \approx k_F^{-1}$). These authors[1,2] have shown that the l dependence of the superconducting correlation length $\xi \equiv \xi(T=0) \approx (\xi_0 l)^{1/2}$ for $k_F l \gg 1$ gives way at the localization threshold to the dependence $\xi \approx (\xi_0 l^2)^{1/3}$ where $\xi_0 = 0.18\, v_F/T_c$. A change of this sort accounts for the fact that at the localization threshold the interval of strong thermodynamic fluctuations near T_c, which is defined by the Ginzburg parameter $t_G = E_F^2/T_c^2 k_F^6 l^6$, is on the order of unity[2] (here $t = |T - T_c|/T_c$).

The results obtained by Bulaevskiĭ and Sadovskiĭ[1] and Kapitulnik and Kotliar,[2] and all the preceding results for dirty superconductors were obtained under the assumption that the superconducting order parameter is self-averaged. In other words, it was assumed that the order parameter, averaged over the impurity configurations, adequately describes the behavior of the system. In order for this assumption to be true, the impurity-induced fluctuations of the order parameter in the sample must be small. We will find below a temperature interval t_D near T_c where this condition is not satisfied. We will show that $t_D \approx t_G^2 \ll t_G$ for $k_F l \gg 1$ and that the assumption that the order parameter is self-averaged far from the localization threshold is correct. At the localization threshold, however, $t_D \gtrsim t_G \approx 1$ because of strong fluctuations of the electronic characteristics near the Anderson transition. Here the equations for the average order parameter cannot be used to describe superconductivity.

To estimate the fluctuation region t_Dm we use the functional for the unaveraged order parameter $\Delta(\mathbf{r})$

$$F_s(\Delta) = \int d\mathbf{r}\,[g^{-1}\,|\Delta(\mathbf{r})|^2 - \int d\mathbf{r}' K(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r})\Delta(\mathbf{r}') + \frac{1}{2} BN(E_F)\,|\Delta(\mathbf{r})|^4\,],$$

$$K(\mathbf{r},\mathbf{r}') = T \sum_{\mu,\nu,n} \varphi_\mu(\mathbf{r})\,\varphi_\mu^*(\mathbf{r}')\,\varphi_\nu(\mathbf{r})\,\varphi_\nu^*(\mathbf{r}')/(i\epsilon_n - \epsilon_\mu)(-i\epsilon_n - \epsilon_\nu), \qquad (1)$$

$$\epsilon_n = \pi T(2n+1),$$

where $\varphi_\mu(\mathbf{r})$ and ϵ_μ are exact intrinsic wave functions and energies of electrons for a given impurity configuration, and $N(E_F)$ is the average density of the electronic states at the Fermi level. The quantities $\varphi_\nu(\mathbf{r})$ and ϵ_ν are random values, so that the kernel $K(\mathbf{r},\mathbf{r}')$, which depends on the arrangement of the impurities, fluctuates in space, causing corresponding fluctuations of the order parameter $\Delta(\mathbf{r})$. We ignore the fluctuations of the effective electron-attraction parameter g and also the coefficient $B(B = 7\zeta(3)/8\pi^2 T^2$.

Assuming that the fluctuations of the kernel are small, we find from (1) the Ginzburg-Landau equation which describes the slow variations of the order parameter $\Delta(\mathbf{r})$ (the fluctuations of the kernel on the scale lower than ξ average out and lead to

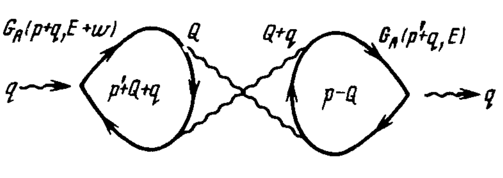

FIG. 1.

only a shift of the transition temperature). The Ginzburg-Landau equation is

$$\left[\frac{T_{c0}-T}{T} + A(\mathbf{r}) - B\,|\Delta(\mathbf{r})|^2 + C\frac{\partial^2}{\partial \mathbf{r}^2}\right]\Delta(\mathbf{r}) = 0,$$

$$C = \frac{1}{6}\int K_0(r) r^2 dr, \quad K_0(\mathbf{r}-\mathbf{r}') = \langle K(\mathbf{r},\mathbf{r}')\rangle ,$$

$$A(\mathbf{r}) = \int_0^{\omega_D} \frac{dE}{E}\tanh\frac{E}{2T_{c0}}\left[\frac{N(\mathbf{r},E)}{N(E_F)} - 1\right], \quad N(\mathbf{r},E) = \sum_\nu |\varphi_\nu(\mathbf{r})|^2\,\delta(E-\epsilon_\nu), \tag{2}$$

where $\langle \cdots \rangle$ denotes averaging over the impurities, $C=\xi^2$, and $N(\mathbf{r},E)$ is the local density of the electronic states. In (2) we have ignored the fluctuations of the coefficient C, and replaced $\int K_1(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}')d\mathbf{r}'$ by $\Delta(\mathbf{r})\int K_1(\mathbf{r},\mathbf{r}')d\mathbf{r}' = A(\mathbf{r})\Delta(\mathbf{r})$, where $K_1(\mathbf{r},\mathbf{r}') = K(\mathbf{r},\mathbf{r}') - K_0(\mathbf{r}-\mathbf{r}')$. The parameter T_{c0} in (2) is the transition temperature in which the long-wave fluctuations of the kernel are ignored.

We see from (2) that in the model under consideration the fluctuations $\Delta(\mathbf{r})$ depend on the distribution of the random $N(\mathbf{r},E)$. For a further analysis we must find the correlation function of this quantity, $S(\mathbf{r},\omega)$, which can be determined in terms of the leading and retarded Green's functions of the electron, $G_{L,R}(\mathbf{r},\mathbf{r}')$, by means of the relation

$$\begin{aligned} S(\mathbf{r},\omega) &= [N(E_F)]^{-2}\langle N(\mathbf{r},E_F+\omega)N(0,E_F)\rangle - 1 \\ &= [N(E_F)]^{-2}\mathrm{Re}\,[\langle G_L(\mathbf{r},\mathbf{r},E_F+\omega)G_R(0,0,E_F) \\ &\qquad - G_L(\mathbf{r},\mathbf{r},E_F+\omega)G_L(0,0,E_F)\rangle] - 1. \end{aligned} \tag{3}$$

Equation (2) with an arbitrary correlation function of $A(\mathbf{r})$ was considered previously[3,4] in a study of the effect of structural inhomogeneities of samples on their superconducting properties. The analysis below is carried out in a similar manner. It follows from Refs. 3 and 4 that the superconducting properties of the system considered depend essentially on the behavior of the correlation function $S(q,\omega)$ at small values of q and ω.

Far from the localization threshold, the function $S(q,\omega)$ for $q \ll l^{-1}$ and $\omega \ll \tau^{-1} = v_F/l$ was found in Ref. 5 for a model of many orbitals at a lattice site. For $k_F l \gg 1$ the same result can be obtained by using a standard procedure of summing a perturbation-theory series over the scattering by impurities for (3). At small values of q and ω, the dominant contribution comes from the diagram in Fig. 1, where the wavy line corresponds to a diffusion, and also from a similar diagram with a cooperon. The function $S(q,\omega)$ is a singular function for small q and ω

$$S(q,\omega) \approx \frac{1}{[N(E_F)]^2 D_0^{3/2}}\,\mathrm{Re}\,\frac{1}{(-i\omega + D_0 q^2)^{1/2}}, \tag{4}$$

where $D_0 = l v_F/3$. The scaling function $S(q,\omega)$ is known at the localization threshold[6]:

$$S(q,\omega) \approx L_\omega^3 F(qL_\omega), \; L_\omega = [\omega N(E_F)]^{-1/3}. \tag{5}$$

This function can also be found from (4) if D_0 is replaced by the effective diffusion coefficient $D(q,\omega) = L_\omega^{-1} f(qL_\omega)$. A further analysis shows that upon integration over q and ω the dominant contribution will come from the region $q < \omega$. In the limit $q/\omega \to 0$ we have $f \approx 1$ and $F(x) \approx (1+x^4)^{-1/4}$.

Using (2), (4), and (5) and carrying out some calculations similar to those in Ref. 3, we find that fluctuations of $N(\mathbf{r},E)$ cause a shift in the transition temperature from T_{c0} to T_c and $T_c > T_{c0}$. Furthermore, the impurities cause the spatial fluctuations of the order parameter to increase as $T \to T_c$. Below T_c the fluctuational contribution due to disorder in the region $k_F l \gg 1$ at $t \ll 1$ is given by

$$\frac{\langle \Delta^2 \rangle - \langle \Delta \rangle^2}{\langle \Delta \rangle^2} = \left(\frac{t_D}{t}\right)^{1/2}, \quad \langle \Delta \rangle^2 = \frac{t}{B}\left[1 - 7\left(\frac{t_D}{t}\right)^{1/2}\right], \tag{6}$$

where $t_D \approx t_G^2 \approx T_c^2/E_F^2 k_F^6 l^6$. The contribution of thermodynamic fluctuations, Δ, to the dispersion is $(t_G/t)^{1/2}$. In a three-dimensional system with $k_F l \gg 1$, the disorder-induced fluctuations are thus considerably weaker than thermodynamic fluctuations.

At the localization threshold below T_c for $t \ll 1$ the quantity $(t_D/t)^{1/2}$ on the right sides of expressions (6) should be replaced by $|\ln t|/\sqrt{t}$. In this region the contribution of thermodynamic fluctuations is on the order of $1/\sqrt{t}$ and, in order of magnitude, $t_D \approx t_G \approx 1$, although there is reason to assume that disorder-induced fluctuations are slightly stronger than thermodynamic fluctuations. In the dielectric region the term $[N(E_F)]^{-1}\delta(\omega)$ should be added to the function $S(q,\omega)$ for small values of $q \ll R_l^{-1}$. Here t_D and t_G have a component which increases approximately as $[N(E_F)T_c R_l^3]^{-2}$ as the localization length, R_l, is decreased. The weak spatial fluctuations ($t_D \ll t_G$) become strong fluctuations, $t_D \approx t_G \approx 1$, when the conductivity is $\sigma \approx \sigma^* \approx \sigma_c \; (k_F \xi_0)^{-1/3}$, where $\sigma_c \approx e^2 k_F/\pi^3$ is on the order of $(2\text{–}5) \times 10^2$ $\Omega^{-1} \cdot \text{cm}^{-1}$.

We see that the impurities cause not only the superconducting correlation length, ξ, to decrease but also account for the spatial fluctuations of the order parameter of the sample, which increase as the Anderson transition is approached. Because of strong spatial fluctuations of the order parameter of the sample over the entire temperature range, a percolation mechanism for the screening and superconducting current flow is typical for a superconductor near the localization threshold.[4] The quasiparticle spectrum in them is apparently gap-free.

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