# Superfluid $^3$He-*A* and the electroweak interaction

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A field theory in superfluid $^3$He-*A* which describes the dynamics of chiral fermion excitations that are interacting with collective boson modes of the order parameter is similar to the theory of the electroweak interaction. The roles of photons and *W* bosons are played by orbital waves and quasi-Goldstone spin-orbit modes, respectively. In $^3$He-*A*, the *W* bosons have a mass because of the strong-coupling corrections, in contrast with the Weinberg-Salam theory, in which they acquire a mass because of the Higgs phenomenon. The effective interaction of fermions with *W* bosons in $^3$He-*A* is of a zero-charge nature and leads to an additional dissipation in the motion of quantized vortices and solitons at low temperatures.

The spectrum of fermion excitations in $^3$He-*A* vanishes at two points on the Fermi sphere: $\mathbf{k} = \pm k_F \mathbf{l}$, where $\mathbf{l}$ is the orbital (liquid-crystal) anisotropy vector. Fermi excitations near these poles play a key role in the dynamics of the liquid at temperatures $T$ low in comparison with the gap amplitude $\Delta_0$, $\Delta(\mathbf{k}) = \Delta_0 |[\mathbf{k},\mathbf{l}]|/k_F$, so that they require systematic study. The interaction of fermions in $^3$He-*A* with the boson fields which characterize the superfluid vacuum [the superfluid velocity $\mathbf{v}_s$, the density $\rho = (1/3\pi^2) k_F^3$, and the vector $\mathbf{l}$] is reminiscent in many ways of quantum electrodynamics.[1,2] Fermions near poles are described by a Dirac equation, and the gauge fields acting on them are

$$\mathbf{A} = k_F \mathbf{l}, \quad A_0 = k_F \mathbf{l} \cdot \mathbf{v}_s . \tag{1}$$

In particular, this analogy makes it possible to determine the mechanism for the transfer of momentum from the vacuum to excitations: The source of the momentum of the excitations, $(k_F \mathbf{l}/8\pi^2) F^*_{\mu\nu} F^{\mu\nu}$, is of the same origin as the anomaly of the chiral current in quantum electrodynamics.[2] In addition, it becomes possible to associate the logarithmic divergence of the Lagrangian for "photons" in $^3$He-*A* (the role of these photons is played by orbital waves: oscillations of the vector $\mathbf{l}$) with the zero-charge phenomenon.

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In Refs. 1 and 2, however, the spin structure of the Fermi excitations and the spin-triplet order parameter of the superfluid state were not taken into account. Incorporating the spin has the consequence of increasing the number of gauge fields acting on the fermions: In addition to the interaction with the field of "photons," there is an interaction with the field of so-called quasi-Goldstone spin-orbit waves,[3] which are completely analogous to $W$ bosons in the theory of the electroweak interaction.[4,5]

An equation of the Dirac type for fermions in $^3$He-$A$ near the poles of the Fermi sphere can be derived from the Bogolyubov equation,[1] in which it is necessary to take into account the fluctuations $\delta A_{\alpha i}$ of the triplet order parameter of the $3\times 3$ matrix $A_{\alpha i}$ around its equilibrium value

$$A^{(0)}_{\alpha i} = \Delta_0 d_\alpha (e_1^i + i e_2^i). \tag{2}$$

The unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ describe the orbital part of the order parameter; their vector product determines the orbital anisotropy vector $\mathbf{l}[\mathbf{e}_1,\mathbf{e}_2]$; and the unit vector $\mathbf{d}$ specifies the spin axis, i.e., the axis of the magnetic anisotropy. Only certain combinations of fluctuations act on the fermions. These combinations form a "photon" field and a $W$ field:

$$A_1 + iA_2 = -\frac{k_F}{\Delta_0} d_\alpha l^i \delta A_{\alpha i},$$

$$W_1^\alpha + iW_2^\alpha = \frac{k_F}{i\Delta_0} e^{\alpha\beta\gamma} d_\beta l^i \delta A_{\gamma i};$$

$$A_3 = \delta k_F, \quad W_3^\alpha = \frac{1}{2k_F} e^{\alpha\beta\gamma} d^\beta \frac{\partial}{\partial t} d^\gamma; \tag{3}$$

$$A_0 = k_F \mathbf{l}\cdot\mathbf{v}_s, \quad W_0^\alpha = \frac{1}{2} k_F e^{\alpha\beta\gamma} d_\beta (\mathbf{l}\cdot\vec{\nabla}) d_\gamma.$$

Equations (3) also incorporate the effect of the fluctuational spin $\mathbf{S}\sim[\mathbf{d}\,\partial\mathbf{d}/\partial t]$, which accounts for a third component of the $W$ field, in precisely the same way as density fluctuations account for a third component, along $\mathbf{l}$, of the "photon" field $\mathbf{A}$.

In terms of the fields in (3), the Bogolyubov equation for the Bogolyubov spinor $\psi$ takes the following form near the poles on the Fermi sphere:

$$\left[\left(i\frac{\partial}{\partial t} - eA_0 - e\sigma^\alpha W_0^\alpha\right) + e(c_\parallel l^i\tau_3 + c_\perp(e_1^i\tau_1 + e_2^i\tau_2))\left(\frac{1}{i}\nabla_i - eA_i - e\sigma^\alpha W_i^\alpha\right)\right]\psi = 0. \tag{4}$$

Here $\tau_i$ and $\sigma^\alpha$ are the Pauli matrices corresponding to the Bogolyubov isospin and ordinary spin. This equation is reminiscent of the Dirac equation for massless chiral fermions in the Weinberg-Salam theory.[4,5] The primary distinction is in the anisotropy of $^3$He-$A$ along the $\mathbf{l}$ and $\mathbf{d}$ axes. The velocity $c_\parallel = v_F$ along $\mathbf{l}$ is far higher than the transverse velocity $c_\perp = \Delta_0/k_F$, and we have $W^\alpha d^\alpha = 0$; i.e., there are no $Z$ bosons. The charge $e = \mathbf{k}\cdot\mathbf{l}/\mathbf{k}_F$ takes on the values $+1$ and $-1$ for fermions near the upper and lower poles, respectively.

The Lagrangian for the "photons" and $W$ field (i.e., for orbital and quasi-Goldstone collective modes) is given in Ref. 3. An important point is that in the approximation of so-called weak coupling (in which the Fermi spheres with different spin projections do not interact with each other) there is an additional SO(3) symmetry, which combines the "photons" and the $W$ bosons in a single triplet (more precisely, a sextet, when we take into account the polarization of the collective modes). In this approximation, the $W$ bosons, like the Goldstone orbital waves (or "photons,") have no mass. The $W$ bosons acquire a mass when the strong-coupling corrections are taken into account; these corrections are small at low pressures. Consequently, and in contrast with the Weinberg-Salam theory, the Higgs phenomenon is not required for the appearance of a mass of the $W$ bosons in $^3$He-$A$.

Without going into detail, we simply note that $^3$He-$A$ also demonstrates how symmetry groups of large dimensionality may arise: In the weak-coupling approximation, the amplitudes of the triplet Cooper pairing, $A_{\alpha i}$, and the amplitude of the singlet pairing, $A_{00}$, form a decuplet which transforms under a representation of the $U(4)$ group. This group breaks spontaneously to the $SO(3)\times U(1)\times U(1)$ group through the formation of a vacuum mean $A^{(0)}_{\alpha i}$, with the result that 11 Goldstones arise: Sound, spin waves (two modes) "photons" (two modes), $W$ bosons (four modes), and $Z$ bosons (two modes of $A_{00}$ fluctuations). The latter, like $W$ bosons, have a mass because of corrections to the weak-coupling approximation.

Yet another distinction between the theories results from the behavior of the effective constant of the interaction of fermions with gauge fields at low frequencies. The charge behaves in the identical zero-charge fashion in the interaction with "photons" and the interaction with $W$ bosons. The reason is that the vacuum in $^3$He-$A$ is an exclusively fermion vacuum, i.e., does not contain additional zero-point vibrations of the free boson fields: Free boson fields, i.e, fields outside the liquid, simply do not exist. Accordingly, the asymptotic freedom for an interaction through $W$ bosons in the electroweak interaction is replaced by a zero-charge behavior in $^3$He-$A$.

The vacuum of superfluid $^3$He-$A$ is diamagnetic with respect to both the "magnetic" field $\mathbf{B} = \mathrm{curl}\mathbf{A}$ and the "color magnetic" field $\mathbf{B}^\alpha$. The energy of field $\mathbf{B}$ can easily be found by summing over Landau levels with the negative energy (Ref. 4, for example). The spectrum of Landau levels for a fermion in magnetic field $\mathbf{B}$ was derived in Ref. 6, with allowance for the anisotropy of the velocity. Using that spectrum, we find the following expression for the "magnetic" energy:

$$F = \frac{\hbar c_\parallel}{24\pi^2} \tilde{B}^2 \ln \frac{\Delta_0^2}{\tilde{B}} \;, \quad \tilde{B}^2 \equiv [\mathbf{B}, \mathbf{l}]^2 + \frac{c_\perp^2}{c_\parallel^2} (\mathbf{B} \cdot \mathbf{l})^2 . \qquad (5)$$

An analogous result is found for the $W$-boson magnetic fields $B^\alpha$. A positive "magnetic" energy (and there clearly can be no negative gradient energy in the stable $A$ phase of $^3$He) means a zero-charge behavior of the effective charge which couples the fermion with both the photon and $W$-boson modes[4]:

$$e^2_{eff} = \frac{3\pi}{\ln \dfrac{\Delta_0^2}{\omega^2}} \; . \qquad (6)$$

The logarithmically divergent part of the Lagrangian for the "photons" and the $W$ field is expressed in terms of $e^2_{\rm eff}$ and in terms of the strength $F_{\mu\nu}$ of the "electromagnetic" field and $F^{(\alpha)}_{\mu\nu}$ of the $W$ field in covariant form

$$\mathcal{L} = \frac{1}{e^2_{eff}} \frac{\sqrt{-g}}{16\pi} (F_{\mu\nu}F^{\mu\nu} + F^{(\alpha)}_{\mu\nu}F^{\mu\nu(\alpha)}); \quad F^{\mu\nu} = F_{\eta\xi} g^{\eta\mu} g^{\xi\nu}, \tag{7}$$

where $g^{\mu\nu}$ is the metric tensor, whose components are, according to (4),

$$g^{00} = -1, \; g^{33} = c^2_{\parallel}, \; g^{11} = g^{22} = c^2_{\perp}, \; \sqrt{-g} = (c_{\parallel} c^2_{\perp})^{-1} . \tag{8}$$

Choi and Muzikar[7] recently found the coefficient of the term $\widetilde{B}^2 \ln \Delta^2_0/\widetilde{B}$ to be zero. The apparent reason for that result is that the discrete nature of the spectrum in the "magnetic" field was ignored.

The interaction of fermions with $W$ bosons in $^3$He-$A$ leads to an additional Schwinger anomaly in the axial current with a source $\sim F^{*(\alpha)}_{\mu\nu} F^{\mu\nu(\alpha)}$, and thus an additional channel for the transfer of momentum from the vacuum to excitations. This circumstance is particularly important in a study of the dynamics of such entities as the interface between the $A$ and $B$ phases of $^3$He (Ref. 8) and quantized vortices in both phases of $^3$He (the gap in the spectrum inside the core of a vortex in $^3$He-$B$ also vanishes at several points on the Fermi sphere[9]) at low temperatures, where the creation of excitations near the poles becomes the sole relaxation mechanism.

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