Metal phonon-spectrum singularities determined by local geometry of the Fermi surface

G. T. Avanesyan, M. I. Kaganov, and T. Yu. Lisovskaya

Institute of Physics Problems, USSR Academy of Sciences
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The existence of singularities in the angular dependence of the speed s and of the absorption coefficient Γ of sound in metal is predicted. The singularities are due to zero-curvature lines (lines of parabolic points) on the Fermi surface. The parabolic points should lead, besides, to enhancement of the Migdal-Kohn singularities and to singularities in the angular dependence of the Pippard (geometric) oscillations of the sound absorption coefficient.

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The Fermi surfaces of most metals are complicated and contain dents, necks, etc. (see, e.g., Appendix III of[1]). We wish to call attention that lines of zero Gaussian curvature (lines of parabolic points) must exist in practice on the Fermi surface, and to indicate the role they play in a number of properties of metals.

It is known[1] that the electrons that take part in the second absorptions are those located on a "strip" whose equation is[2]

$$\mathbf{k}\,\mathbf{v} = \omega; \qquad \epsilon(\mathbf{p}) = \epsilon_F . \tag{1}$$

Here ω is the frequency, k is the wave vector of the sound wave, $s=\omega/k$ is the speed of sound, p is the quasimomentum, ϵ is the energy, $\mathbf{v}=\partial\epsilon/\partial\mathbf{p}$ is the velocity, and ϵ_F is the electron Fermi energy. We assume that $kl \gg 1$, where l is the electron mean free path. Since $v_F \gg s$, the vectors $\mathbf{v}$ and $\mathbf{k}$ are almost perpendicular. If the Fermi surface contains a line of parabolic point, then the topology of the strip (1) must of necessity change at certain values[1)] of $\mathbf{n}=\mathbf{k}/k$ whenever the direction of sound propagation changes. We shall designate these values by $\mathbf{n}_C$. It can be shown that at $\mathbf{n}=\mathbf{n}_C$ two cases are possible: either a) the strip vanishes (appears), or b) the strip breaks. Each critical direction n_C corresponds to a definite parabolic point (point A). Near the point A the Fermi surface is for the form shown in Fig. 1. The structure of the strip at $\mathbf{n}\approx\mathbf{n}_C$ (near the point A) is shown in Fig. 2. In the first case (Fig. 2a) the

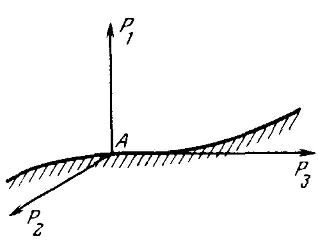

FIG. 1.

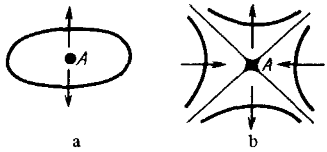

FIG. 2.

critical value $\mathbf{n}=\mathbf{n}_C$ corresponds to degeneracy of the strip into a point ($p_1=p_2=0$), and in the second (Fig. 2b) the strip interesects itself at $\mathbf{n}=\mathbf{n}_C$. This accounts for all the topological singularities of the strip, if we assume no special (accidental) properties of the Fermi surfaces.

The existence of singularities on the strips leads to singularities in the dependence of the coefficient of sound absorption by the electrons on the direction $\mathbf{n}$ of the sound. They can be easily calculated by analyzing the expression for the absorption coefficient

$$\Gamma = \int |\Lambda|^2 \delta(\epsilon - \epsilon_F)\, \delta(s - \mathbf{n}\mathbf{v})\, d^3p, \tag{2}$$

Λ is the matrix element of the electron-phonon interaction and includes all the factors (see, e.g.,[1] Appendix II). In the first case (Fig. 2a) at $\mathbf{n}=\mathbf{n}_C$ the coefficient Γ experiences a finite discontinuity $\delta\Gamma \sim \Gamma_0$, where $\Gamma_0 \approx \sqrt{m/M}(sk)$ is of the order of magnitude of the coefficient of absorption by the large Fermi surface (cf.[3]). Here m and M are the masses of the electron and ion. In the second case (Fig. 2b), Γ has a logarithmic singularity $\delta\Gamma \sim \Gamma_0 \ln|\mathbf{n}-\mathbf{n}_C|$. Actually the jump and the logarithmic singularities become smoothed out. The amount of smoothing is determined by the largest of the three parameters $1/kl$, T/ϵ_F, $\hbar\omega/\epsilon_F$ (T is the temperature). The singularities of $\Gamma = \mathrm{Im}\omega$, naturally, are accompanied by a singularity in $\mathrm{Re}\omega$, and the jump of Γ corresponds to a logarithmic singularity of $\mathrm{Re}\omega$, while a logarithmic singularity of Γ corresponds to a jump of $\mathrm{Re}\omega$.

Since every Fermi surface has an inversion center, each parabolic point has an "antipode" with $\mathbf{v}=-\mathbf{v}_A$. There must therefore exist a critical direction n_C' close to n_C and having analogous singularities (see Fig. 3a and Fig. 3b). Since $\Gamma_0 \propto sk$, the singularities listed above must be treated as singularities of the speed of sound s.

The structure of the expression for the conductivity tensor $\sigma_{ik}(k)$ at $kl \gg 1$ (the asymptotic value of σ_{ik} determines the value of the impedance under conditions of the anomalous skin effect) is similar to the structure of expression (2) for the absorption coefficient Γ. Therefore the conductivity component σ_{11} should also have the singularities described above. The other components σ_{ik} have much weaker singularities, since $v_2=v_3=0$ at the point A. The parabolic points of the Fermi surface can serve as sources of anomalies in those effects which are due to electrons that belong to a small vicinity near the chosen point. All the effects due to the limiting points of the Fermi surface should be sensitive in this sense. We present two examples.

1. *There should exist cones of directions in which the Migdal-Kohn singularities are enhanced* (see[4]). The enhancement takes place when the vector $\hbar\mathbf{k}$ *is equal to the limiting diameter joining the point A with its "antipode."*

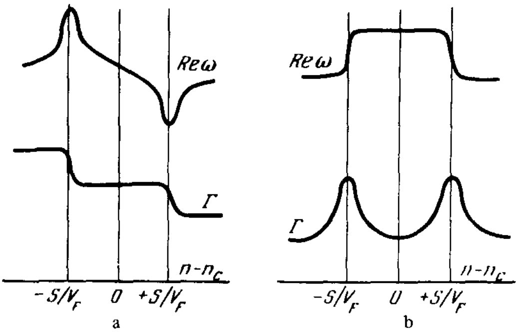

FIG. 3.

2. Breaking of the belt (1) should lead to a jumplike change in the periods of the Pippard (geometric) oscillations of the sound absorption[5] in a magnetic field, and at the "instant" of the break (at $\mathbf{n}=\mathbf{n}_C$) the amplitude of the oscillations increases by a factor $(k\tau_H)^{1/6}$ if the extremal trajectory of electron motion in the magnetic field contains one parabolic point, and by a factor $(k\tau_H)^{1/3}$ if there are two such points.

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1)Related problems were considered by V. M. Kontorovich and N. A. Sapogova (oral communication).

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