Mesons from gluons

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The mass of the meson (gluonium) that contains no quarks and consists of gluons only is calculated within the framework of the MIT bag model. We consider hadrons that contain valence gluons besides quarks.

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The presently discussed models of strong interactions start with the assumption that there exist, besides the quarks, also boson gluon fields. This raises the question of the existence of mesons that do not contain quark-antiquark pairs and consist of gluons only. The possibility of the existence of a meson of this type (the so-called gluonium) has been discussed in the literature from various points of view.[1–6]

There are at present serious grounds for assuming that the so called standard model (SM) of hadrons is valid. According to this model, hadrons consist of colored quarks, the interaction between which is described by colored Yang-Mills (YM) fields. The corresponding theory has been dubbed quantum chromodyanmics (QCD) in analogy with quantum electrodynamics (QED). In QCD, the gluon state of the simplest type—a quantum of the octet colored YM field—cannot exist in accordance with the assumption made in the SM, namely that the observed spectrum contains only white states. Therefore the simplest white state of gluonium is a system of two gluons.[2] We calculate below the mass of such a state in the MIT bag model.[7–10] This model can be regarded as a phenomenological quantitative description of a system of trapped colored quarks and gluons and gives good results for low-lying hadron states of the ordinary type. One can therefore hope that the results that follow for gluonium will also turn out to be close to reality.

We consider two valent gluons "trapped" in a bag of radius R. The interaction between the gluons contains a small constant [11] $g^2/4\pi = 0.23$, which will henceforth be neglected. In the zeroth approximation, the valence gluons will then behave like an octet of massless Abelian fields that satisfy Maxwell's equations and the conditions that they do not escape

$$n_\mu F^a_{\mu\nu} = 0, \quad (r = R), \qquad (a = 1, 2, \ldots, 8). \tag{1}$$

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The mass of gluonium is given by

$$M = \frac{2x_g}{R} + BV - \frac{Z_0}{R} . \tag{2}$$

Here V is the volume of the bag, while the constants B and Z_0 are the principal parameters of the MIT model, equal, according to the parametrization of[11], to

$$B^{1/4} = 0.169\,\text{GeV}, \qquad Z_0 = 1.65.$$

The first term in (2) is the sum of gluon energies, each of which is equal to x_g/R. To find M, we calculate the minimal x_g compatible with (1), and minimize (2) with respect to R. Then

$$M = \frac{4}{3}(4\pi B)^{1/4}(2x_g - Z_0)^{3/4} . \tag{3}$$

The states of the gluon inside the bag, as well as the states of the photon, are described by longitudinal and transverse spherical vectors.[12] For a gluon of the magnetic type with angular momentum j, the "electric" and "magnetic" fields are given by

$$\mathbf{E}^a_M = C j_j(\omega r)\, \mathbf{Y}_{jjm}(\mathbf{n})\, \chi^a ,$$

$$\mathbf{B}^a_M = C\left[\sqrt{\frac{j}{2j+1}}\, j_{j+1}(\omega r)\, \mathbf{Y}_{j,j+1,m}(\mathbf{n}) - \sqrt{\frac{j+1}{2j+1}}\, j_{j-1}(\omega r)\mathbf{Y}_{j,j-1,m}(\mathbf{n})\right]\chi^a. \tag{4}$$

For gluons of the electric type we have $\mathbf{E}^a_E = -\mathbf{B}^a_M$ and $\mathbf{B}^a_E = \mathbf{E}^a_M$. Here $j_j(\omega r)$ are spherical Bessel functions, ω is the gluon energy, $\mathbf{Y}_{jlm}$ are spherical vectors, χ^a is the colored wave function of the gluon, and C is a normalization coefficient. In terms of the fields $\mathbf{E}^a$ and $\mathbf{B}^a$, the boundary condition (1) takes the form

$$\mathbf{n} \times \mathbf{E}^a = 0, \tag{5a}$$

$$\mathbf{n} \times \mathbf{B}^a = 0. \tag{5b}$$

For gluons of the magnetic type with angular momentum $j=1$, these conditions lead to the following equation for $x=\omega R$:

$$\operatorname{ctg} x = \frac{1}{x} - x . \tag{6}$$

For gluons of the electric type with $j=1$, we have

$$\operatorname{tg} x = x . \tag{7}$$

The first positive root of Eq. (6) for magnetic gluons is $x = x_g = 2.743$. The positive roots of Eq. (7) lie higher than this value. States with angular momenta $j>1$ also correspond to higher values of R. Therefore the state of a magnetic gluon with energy $\omega = x_g/R$ and with angular momentum $j=1$ are ground states. Numerically, the mass of the ground state of gluonium is, according to (3):

$M = 1.15\,\mathrm{GeV}$.

This state has quantum numbers $J^{PC} = 0^{++}$. It is assumed here that the bag itself has the quantum numbers of vacuum.

We consider next mesons made up of quarks and one gluon $q\bar{q}g$ (g stands for the gluon). As shown in[2], there are no analogous white $qqqg$ baryons if the quarks are in the ground s state. It is clear that among the $q\bar{q}g$ mesons there are particles that have the same quantum numbers as the "ordinary" $q\bar{q}$ mesons. Transitions $q\bar{q}g \to q\bar{q}$ occur between them, and the amplitudes of these transitions are linear in the quark-gluon interaction constant g. Taking these transitions into account, the masses of the diagonal states of the Hamiltonian are equal to

$$\mu_1 = m_1 - \frac{m_{12}^7}{m_2 - m_1}, \qquad \mu_2 = m_2 + \frac{m_{12}^2}{m_2 - m_1}.$$

Here m_1 and m_2 are the masses of the $q\bar{q}$ and $q\bar{q}g$ mesons in the zeroth order in the constant g, while m_{12} is the matrix element of the transition $q\bar{q} \to q\bar{q}g$. It is seen therefore that the existence of states $q\bar{q}g$ in the presence of transitions $q\bar{q} \to q\bar{q}g$ leads to the appearance of corrections to the spectrum of the light mesons, and these corrections are proportional to g^2. We recall that allowance for the one-gluon exchange between the quarks at short distances also leads to corrections of the order of g^2.

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