Resistance of *N-S* interface at low temperatures

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The scattering of electrons at an interface of pure metals causes the resistance of an *N-S* boundary to grow as the temperature is lowered. The resistance of a tunneling contact at $T = 0$ is finite owing to two-particle processes.

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The experimental data on the electrical resistance $R_{NS}(T)$ of the interface between a normal (N) and a superconducting (S) metal are in good agreement with the theory at temperatures in the immediate neighborhood of the critical temperature T_c. At lower temperatures a nonmonotonic temperature dependence of R_{NS} has frequently been observed[1,2] in samples of pure metals, the behavior of the resistance being very sensitive to the method used to prepare the samples. This latter fact indicates the possible presence of a barrier (scattering layer) at the interface of the metals, formed by a thin dielectric layer or a large concentration of structural defects at the interface. In this letter it is shown that even a small amount of electron scattering at the boundary between pure metals alters the temperature dependence of the resistance. The resistance $R_{NS}(T)$ is evaluated for low temperatures and for arbitrary transmission of the barrier.

The resistance of an *N-S* interface is determined by the processes which transform the normal current into superconducting current. Of these processes, the most important in the temperature region $T < \Delta$ (Δ is the order parameter) is Andreev reflection.[3] We shall examine how the finite penetrability of the boundary affects the probability of this process. Suppose that the N metal occupies the region $x < 0$, and the S metal occupies $x > 0$. At the boundary there is a barrier whose thickness is small compared to the coherence length ξ (0). In the quasiclassical approximation $p_F \xi(0) \gg 1, \hbar = 1$), the excitation states can be classified according to the direction of the momentum $\mathbf{p} = p_F \mathbf{n}$.[3] This corresponds to representing the wave function in the form $\psi_n \exp(i p_{Fz} nr)$, where ψ_n is a slowly varying function which, in the absence of magnetic field, satisfies the relation[3]

$$(-i v_F \sigma_z \mathbf{n} \nabla + \Delta \sigma_x) \hat{\psi}_{\mathbf{n}} = \epsilon \hat{\psi}_{\mathbf{n}}, \quad \hat{\psi} = \begin{pmatrix} u \\ v \end{pmatrix} \tag{1}$$

where ϵ is the excitation energy, and σ_x and σ_z are Pauli matrices. Scattering at a metallic interface is accompanied by a change in $\mathbf{n}$, and therefore Eq. (1) is not satisfied in the immediate vicinity of the interface. We assume for simplicity that the scattering causes specular reflection, i.e., only transitions between states with $\mathbf{n}$ and $\mathbf{n}^*$ related by $n_x^* = -n_x, n_{y,z}^* = n_{y,z}$. The effect of the barrier can be taken into accoung by means of the boundary condition ($n_x > 0$)

$$\hat{\psi}_{\mathbf{n}}(+0) = S\hat{\psi}_{\mathbf{n}}(-0) + R'\hat{\psi}_{\mathbf{n}^*}(+0), \quad \hat{\psi}_{\mathbf{n}^*}(-0) = R\hat{\psi}_{\mathbf{n}}(-0) + S'\hat{\psi}_{\mathbf{n}^*}(+0), \tag{2}$$

where S (S') and R (R') are the amplitudes of the transmitted and reflected waves for incidence of an electron at the Fermi energy from the left (right). These are related by $S' = S$, $R' = -R^*S/S^*$, $|S|^2 + |R|^2 = 1$ We assume that $\Delta = 0$ in the N region and that $\Delta(x) = \Delta_0(T)$ for $x \gtrsim \xi(T)$. In these regions Eq. (1) has both electron and hole solutions. The direction of motion of an electron with momentum $p_F\mathbf{n}$ is along $\mathbf{n}$; of a hole, along $-\mathbf{n}$. In Fig. 1 the directions of propagation of particles with momenta $p_F\mathbf{n}$ and $p_F\mathbf{n}^*$ are shown by arrows. Solving Eqs. (1) and (2) simultaneously, we obtain the scattering matrix S_{fi} ($f,i = 1, 2, 3, 4$). i.e., a relation between the amplitudes of the normalized flux of states corresponding to motion toward the interface (index i) and away from the interface (index f). The labeling of the states is explained in Fig. 1. the unitary matrix S_{fi} is of the form

$$\hat{S} = (1 - \alpha^2|R|^2)^{-1} \begin{pmatrix} |S|^2\alpha & R'S^*\alpha\beta & S^*\beta & R^*(1-\alpha^2) \\ -RS^*\alpha\beta & \beta(\alpha|R|^2 - \alpha^*)/\beta^* & R'^*\beta^2 & S^*\beta \\ S\beta & R'\beta^2 & \beta(\alpha|R|^2 - \alpha^*)/\beta^* & R'S^*\alpha\beta \\ R(1-\alpha^2) & S\beta & -RS^*\alpha\beta & |S|^2\alpha \end{pmatrix} \tag{3}$$

where α and β are functions of ϵ and $\mathbf{n}$, $|\alpha|^2 + |\beta|^2 = 1$ Here α is the amplitude of a reflected hole, and β is the amplitude of a transmitted electron for incidence of an electron from the N metal in the absence of a barrier.

The quantity $|S_{11}|^2$ is equal to the probability of Andreev reflection for an elec-

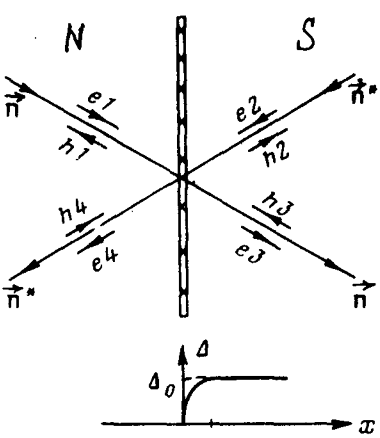

FIG. 1. Scattering at an N-S boundary with a barrier. The letters labeling the arrows directed away from the boundary denote the type of excitation (e for electron, h for hole) formed in the collision with the boundary of the particles labeled by letters near the arrows directed toward the boundary. The numbers labels the initial and final states.

tron incident from the N metal. It follows from Eq. (3) that $|S_{11}|^2 \sim |S|^4$, i.e., to the square of the transmittance of the barrier. This is a natural consequence of the two-particle character of Andreev reflection. For energies $\epsilon > \Delta_0$ the transfer of charge from N to S can occur through injection of excitations. Equation (3) implies that the probability of such a process is proportional to $|S|^2$. Near $T_c(\epsilon \sim T \gg \Delta_0)$ the most important charge transfer processes are the single-particle processes, while for $T < \Delta_0$ it is the two-particle processes that are most important. The decrease in the effective transmittance as the temperature is lowered leads to an increase in the resistance of the boundary.

Using Eq. (1), one can evaluate α for $\epsilon \ll \Delta_0$ and arbitrary dependence of $\Delta(x)$:

$$a(\epsilon, n) = -i(1 - \epsilon^2/2\delta_n^2) + \epsilon/\delta_n \; ,$$
$$\delta_n^{-1} = 2 v_F^{-1} \int_0^\infty dx \exp[-2 v_F^{-1} \int_0^x dy \, \Delta(zy)] \; , \qquad (4)$$

where z is the cosine of the angle of incidence of the electron. If $T \ll \Delta_0$, the energy region $\epsilon > \Delta_0$ makes an exponentially small contribution. Solving the kinetic equation in the normal metal $(\epsilon \sim T \ll \Delta_0)$ with the condition that at the interface

$$f^e_{n^*} = |S_{41}|^2 f^e_n + |S_{44}|^2 f^h_{n^*} \; ,$$

where f^e and f^h are the distribution functions of electrons and holes, and taking Eq. (4) into account, we find for the resistance of the N-S interface:

$$R_{NS}^{-1}(T) = (\rho l)^{-1} (3\overline{|S|^4}/8 \, \overline{|R|^2}) (1 + \pi^2 T^2 / 3\overline{\delta^2}) \; ,$$
$$\overline{|R|^2} = \int_0^1 dz \, z^2 12 \, |R|^2 / (|S|^4 + 4|R|^2) ,$$
$$\overline{|S|^4} = \int_0^1 dz \, 8z \, |S|^4 / (|S|^4 + 4|R|^2) \; , \qquad (5)$$
$$\overline{(\delta^2)}^{-1} = (\overline{|S|^4})^{-1} \int_0^1 dz \, 8z \, |S|^4 / \delta_n^2 (|S|^4 + 4|R|^2) ,$$

where ρ and l are the resistivity and mean free path of the normal metal. Equation (5) is valid for $T \ll \Delta_0$ and $|S|^2 \gg \exp(-\Delta_0/T)$. If the latter condition is violated, single-particle excitation-injection processes are important. Taking these processes into account reduces to adding to the right-hand side of Eq. (5) the well-known expression for the conductivity of an N-S interface evaluated with the tunneling Hamiltonian.[4] For interfaces with a small barrier transmittance (tunneling), a deviation from the theory based on first-order perturbation theory arises in the temperature region $T \sim \Delta_0/\ln|S|^{-2}$, where $|S|^2 \cong \rho l / R_{NS}(T_c$ The resistances of the tunneling contact at $T = T_c$ and $T = 0$ are related by

$$R_{NS}(0) / R_{NS}(T_c) \lesssim 3 R_{NS}(T_c) / 2\rho l$$

where the equal sign obtains for the case of an interface with a uniform barrier.

Using the kinetic equation and the scattering probabilities (3), one can find the resistance of an interface for arbitrary temperature and barrier penetrability if values of $\alpha(\epsilon,\mathbf{n})$ [the function $\Delta(x)$] are known. We will not dwell on this question here. We only mention that the simplest model $\Delta(x) = \Delta_0$ leads to fair agreement with the data of Refs. 1 and 2. For interfaces with a pronounced nonmonotonic behavior of $R_{NS}(T)$ (Ref. 1), the barrier transmittance $|S|^2$ is around 0.1 to 0.5.

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