On the possibility of exciting zero sound in He^3 at speeds below the Fermi speed

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It is shown that a large-amplitude wave can propagate in a Fermi liquid at a speed that is less than the Fermi speed. The collisionless damping in this case is proportional to the small parameter $a=(\omega_0\tau)^{-1}$.

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In the Landau theory of a Fermi liquid it is assumed that zero-sound excitations with a speed w that is greater than the Fermi speed v_F are not subject to collisionless damping. Excitations with speeds w less than v_F are strongly damped. In the linear theory the collisionless damping is weak only for transverse modes whose speed is close to v_F.

In this letter it is shown that , in addition to the usual zero sound, a large-amplitude wave can propagate in a Fermi liquid at a speed less than the Fermi speed; in the field of such a wave there is trapping of resonant quasiparticles. In this case the collisionless damping is proportional to the small parameter $a=(\omega_0\tau)^{-1}$, where $\omega_0=k\sqrt{\phi_0/m}$ is the vibrational frequency of the trapped particles, τ is the relaxation time, and ϕ_0 is the amplitude of the local energy.

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To calculate this effect, one uses the Boltzmann equation to find the distribution functions of the trapped and untrapped particles. We seek a total distribution frunction of the form

$$n = n_0(\epsilon_{\mathbf{p}}) + \frac{\partial n_0}{\partial \epsilon}\delta\epsilon_{\mathbf{p}} + g(\mathbf{p},\mathbf{r},t),$$

$$\delta\epsilon_{\mathbf{p}}(\mathbf{r},t) = \sum_{\mathbf{p}'} f_{\mathbf{pp}'}\delta n_{\mathbf{p}'}(\mathbf{r},t), \tag{1}$$

where $n_0(\epsilon_{\mathbf{p}})$ is the equilibrium distribution function, $\delta\epsilon_{\mathbf{p}}(r,t)$ is the local energy of the quasiparticles, $\delta n_p = \frac{\partial n_0}{\partial \epsilon}\delta\epsilon_{\mathbf{p}} + g$, and $f_{\mathbf{pp}'} = f_0$ is a function describing the interaction of the quasiparticles. Here the function $g(\mathbf{p},\mathbf{r},t)$ satisfies the kinetic equation

$$\frac{\partial g}{\partial t} + \mathbf{v}\frac{\partial g}{\partial \mathbf{r}} - \frac{\partial \delta\epsilon}{\partial \mathbf{r}}\frac{\partial g}{\partial \mathbf{p}} = -\frac{g}{\tau} - \frac{\partial n_0}{\partial \epsilon}\frac{\partial \delta\epsilon}{\partial t}. \tag{2}$$

The collision integral was chosen in the simplest form: $I\{n\} = -g/\tau$.

Using the above expressions for the distribution functions of the trapped and untrapped particles, one can show that under the condition $\delta\epsilon/\epsilon_F \ll 1$ the main contribution to the energy in Eq. (1) is from the large group of nonresonant particles. The distribution function of these particles has the same form as in the linear theory. For this reason, the local energy in Eq. (2) can, as in the Landau theory, be considered a harmonic function of time and the coordinates:

$$\delta\epsilon = -\phi_0 \cos\xi \tag{3}$$

where $\xi = kz - \omega t$, k is the wave vector, and ω is the frequency of the wave. The correction to the energy from Eq. (3) is proportional to the small parameters ϕ_0/ϵ_F and a.

Solving Eq. (2) with the local energy (3) by the method of characteristics, we find the distribution functions for the trapped g_t and untrapped g_{ut} particles in the resonance region as

$$g_t = \phi_0 \frac{w}{\bar{v}}(a\xi - s)\frac{\partial n_0}{\partial \epsilon}, \quad |s| \leqslant 2\cos\frac{\xi}{2}, \tag{4}$$

$$g_{ut} = \phi_0 \frac{w}{\bar{v}}[a(\xi - \bar{\xi}) - (s - \bar{s})]\frac{\partial n_0}{\partial \epsilon}, \quad |s| \geqslant 2\cos\frac{\xi}{2}, \tag{5}$$

where $s = \frac{v_x - w}{\bar{v}}$ is the dimensionless speed, $\bar{s} = \frac{\pi}{\kappa K(\kappa)}$ is the average speed of the trapped particles, $\bar{\xi} = \frac{\pi F\left(\frac{\xi}{2},\kappa\right)}{K(\kappa)}$, κ is the integral of motion, defined by the relation

$\frac{s^2}{2} - \cos\xi = \frac{2}{\kappa^2} = 1$, $K(\kappa)$ is a complete elliptic integral of the first kind, $F\left(\frac{\xi}{2},\kappa\right)$ is an incomplete elliptic integral of the first kind, and $\tilde{v} = \sqrt{\phi_0/m}$. In the nonresonant region the distribution function can be found in the usual way by iteration with respect to ϕ_0.

If the wave is weakly damped, one can obtain the damping coefficient in the linear theory of Fermi liquids by equating the rate of change of the total energy of the nonresonant particles to the work done on them by the wave per unit time. The damping coefficient obtained in this way is the same as one gets from the linear dispersion relation. In the nonlinear region ($\omega_0\tau \gg 1$) also we find the damping of the wave (assuming it is small) by the law of conservation of energy. The rate of change of the energy of the nonresonant particles will be the same here as in the linear theory, but the work done by the wave on the resonant particles will have to be recalculated. The change in the total energy of the nonresonant particles in the field of the wave will be found from the expression:

$$\langle \Delta E \rangle = \langle \sum_{\mathbf{p}} \epsilon_{\mathbf{p}}\, \delta n_{\mathbf{p}} \rangle + \frac{1}{2} \langle \sum_{\mathbf{p}\mathbf{p}'} f_{\mathbf{p}\mathbf{p}'}\, \delta n_{\mathbf{p}}\, \delta n_{\mathbf{p}'} \rangle . \tag{6}$$

Here the brackets denote averages over the wavelength λ. To determine the kinetic energy in Eq. (6), it is necessary to find $\langle \delta n_{\mathbf{p}} \rangle$ to second order in ϕ.[1] The work done by the wave on the resonant particles can be obtained from the expression

$$\langle j_{\rm res} \left(-\frac{\partial \delta\epsilon}{\partial z}\right) \rangle = -\frac{2}{\lambda} \int_0^{\lambda} dz \int \frac{\partial \delta\epsilon}{\partial z} v_z\, \delta n_{\rm res} \frac{d\mathbf{p}}{(2\pi\hbar)^3}, \tag{7}$$

where $j_{\rm res}$ is the resonant-particle current. We define the damping coefficient by the relation

$$\frac{d}{dt} \langle \Delta E \rangle = -2\gamma \langle \Delta E \rangle . \tag{8}$$

Using Eqs. (4) and (5) to evaluate (8), and the distribution function of the nonresonant particles to order ϕ_0^2 to determine the change in the energy (7), we obtain

$$\gamma_N = a\,\omega \frac{\left[\frac{64}{9} + 16 \int_0^1 \frac{d\kappa}{\kappa^4} \left(2E(\kappa) - \frac{\pi^2}{2K(\kappa)}\right)\right]}{\pi \left.\frac{dW(s)}{ds}\right|_{s_0}} \approx 2a\,\gamma_L \tag{9}$$

Here γ_L is the linear damping coefficient for a weak wave, $s_0 = w/v_F$ is the relative speed of the wave, and $W(s) = \frac{s}{2}\ln\left|\frac{1+s}{1-s}\right| - 1$. One can find s_0 from the linear dispersion relation for $a \ll 1$, in which case the damping is small:

$$W(s) = 1/F_0 . \tag{10}$$

A numerical calculation gives $s_0 = 0.85$ for $F_0 = 10.8$

The calculation of the work done on the particles by the wave is analogous to the corresponding problem in the theory of nonlinear damping of electromagnetic and sound waves in metals. The result $\gamma_N = 2a\gamma_L$ was obtained in Ref. 1 for the case $a \ll 1$. It should be pointed out that Eq. (9) is valid only in order of magnitude, since an approximate expression for the collision integral was used in the kinetic equation.

In a smiliar way it can be shown that in a Fermi-liquid model with $f_{\mathbf{pp'}} = f_0 + f_1 \cos\theta$ the collision-damping coefficient of the $m = 0$ and $m = 1$ modes, which have a speed $w < v_F$, is smaller in the highly nonlinear regime than in the linear regime by a factor of a.

Numerical estimates show that nonlinear effects become important in He^3 at temperatures of the order of 10^{-3} K for power flux densities in the quartz emitter of the order of 10^2 W/cm^2. In this case the parmeter $\omega_0\tau$ is about 10^2.

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[1)]For this it is sufficient to use Eq. (1) for $\delta\epsilon_{\mathbf{p}}(\mathbf{r},t)$. Allowance for terms of higher order in $\delta n_{\mathbf{p}}$ in the expansion of $\delta\epsilon_{\mathbf{p}}$ does not affect the values of $\langle\delta n_{\mathbf{p}}\rangle$ to second order.

[1]G. A. Vugal'ter and V. Ya. Demikhovskii, Pis'ma Zh. Eks. Teor. Fiz. 22, 454 (1975) [JETP Lett. 22, 219 (1975)].