Coherence and magnetic-bremsstrahlung effects in radiation by relativistic particles moving in a crystal near a crystallographic axis

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(Submitted 15, May 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 2, 179–182 (20 July 1980)

It is predicted that the intensity of radiation from relativistic particles moving in a crystal at energies exceeding the barrier energy (superbarrier particles) will be much higher than that from particles moving in an amorphous medium. A number of results obtained in experiments on radiation by channeled particles are explained. New experiments are proposed to detect the predicted effect.

PACS numbers: 61.80.Mk

It is predicted that the intensity of radiation by particles moving in a crystal at energies exceeding the barrier energy (superbarrier particles) will be much higher than that radiated by particles moving in an amorphous medium. An experiment to detect this effect is proposed.

1. In the motion of fast particles in a crystal near one of the crystallographic axes (here the z axis), there is always, in addition to channeled particles, a significant number of superbarrier particles—particles executing chaotic motion in a plane orthogonal to the z axis.

In this letter it is predicted that the intensity of the radiation by this group of superbarrier particles will be significantly higher than that of particles moving in an amorphous medium. It is shown that in the low–and high-frequency regions this effect is due to the coherent and magnetic–braking mechanisms of radiation by particles in crystals. On the basis of the results obtained, an explanation is given for a number of the experimental results obtained at Stanford,[1] Khar'kov,[2] and Tomsk[3] in studies of radiation by channeled particles. New experiments are proposed which would be capable of distinguishing the predicted effect against the background of the other mechanisms by which particles radiate in crystals.

2. We shall examine the process of radiation by relativistic particles in the case in which the beam incident on the crystal enters at an angle θ to the crystallographic axis z that is comparable to the critical angle for channeling, θ_c.[4] In this case all of the particles execute a chaotic motion in the crystal in the plane orthogonal to the z axis. Therefore, the intensity of radiation can be related to the energy radiated by a particle in the frequency interval $(\omega,\omega + d\omega)$ for an individual chain of atoms in the crystal by the equation

$$I' = dI/d\omega = 2N_c\,\theta d\int_0^\infty db\,E'(b)\,, \tag{1}$$

where d is the distance between atoms in the chain, b is the impact parameter of the chain,[5] and N_c is the number of atoms in the crystal.

In this problem, the low- and high frequency regions are characterized by different relations between the coherence length $l = \delta^{-1}\max[1,(\gamma\theta)^{-2}]$ and the size of the spatial region $r = 2R/\max(\theta,\theta_c)$ in which the trajectory of the particle in the field of the chain of atoms changes appreciably; here $\delta = \omega/2\gamma^2$, γ is the Lorentz factor of the particle, θ is the scattering angle for scattering within the coherence length, and R is the screening radius of the atom. The mechanism by which relativistic particles radiate in a crystal is different in each of these cases, so we shall examine the high- and low-frequency regions separately.

3. In the frequency region for which $l \ll r$, the radius of curvature of the trajectory of the particle changes little over the coherence length, and therefore one can find $E'(b)$ in the high-frequency region by the methods of the theory of magnetic bremsstrahlung.[7] Taking it into account that the energy of the particle is conserved in the plane orthogonal to the axis of the chain, we find that

$$I' = N_c\frac{4e^2}{\sqrt{3}}\,\delta d\int_{\rho_0}^\infty \rho\,d\rho\int_{\omega/\omega_c}^\infty dx\,K_{5/3}(x), \qquad \omega_c = \frac{3}{2m}\,\gamma^2|\nabla U(\rho)|\,, \tag{2}$$

where $U(\rho)$ is the continuous potential of the chain[4], $K_{5/3}(x)$ is a modified Bessel function, and m is the electron mass. For positrons, the quantity ρ_0 is determined from the condition $\epsilon\theta^2/2 = U(\rho_0)$; for electrons, $\rho_0 = 0$.

It follows from Eq. (2) that for electrons the intensity of radiation in the high-frequency region is independent of θ. As θ increases, however, the domain of applicability of Eq. (2) rapidly diminishes.

For electrons in the frequency region $\omega > \omega_c$ the main contribution to the integral in Eq. (2) is from the region $\rho \lesssim R$, in which $U(\rho) \simeq U_0\ln(R/\rho)$.[4] In this case the intensity of radiation by the electrons is given by

$$I' = N_c 2\pi U_0^2 d/3m^2\delta \ . \tag{3}$$

If a parallel beam of positrons is incident on the crystal along the z axis, the radiation of each of the particles will depend on the position ρ_0 of its entrance into the crystal. The intensity of radiation (2) in this case should be averaged over ρ_0. If the potential of the chain is approximated by a potential of the form $U(\rho) = U_0(1 - \rho/r)$ for $\rho \leqslant r$ and $U(\rho) = 0$ for $\rho > r$, then

$$I' = N_c \frac{\pi\sqrt{3}e^2}{4m^2} U_0^2 nd^2 r^3 F(\zeta) , \qquad \zeta = \frac{4}{3}\frac{\delta mr}{U_0} , \tag{4}$$

where n is the density of atoms and $F(\zeta)$ is a function which is widely used in the theory of synchrotron radiation (see Fig. 17 of Ref. 7). According to the Eq. (3) the intensity at the maximum of the spectrum should exceed the intensity of radiation by relativistic particles in an amorphous medium by about a factor of $(mndr^3/10Ze^2)$, where Ze is the nuclear charge of the atom $(U_0 \simeq Ze^2/d)$.

Formulas (3) and (4) give a good discription of the experimental data of Refs. 1–3, which pertain to the high-frequency region. This indicates that in these experiments the radiation in a wide region of the spectral distribution is goverened not by channeled, but by superbarrier particles.

4. If $l \gg r$, then a substantial change in the trajectory of the particle occurs with in the radiation-forming zone. In this case $E'(b)$ can be found by the methods of the theory of radiation by relativistic particles in the field of atomic nuclei.[6] Using the equation of motion of the particle in the field of the chain of atoms along with Eq. (35) of Ref. 6, we find that

$$I' = N_c \frac{4e^2}{\pi} \theta d \int_0^\infty db \left[\frac{2\xi^2 + 1}{\xi\sqrt{\xi^2 + 1}} \ln\left(\xi + \sqrt{\xi^2 + 1}\right) - 1 \right] , \tag{5}$$

where $\xi = \gamma\theta(b)/2$ and $\theta(b)$ is the scattering angle for scattering of the particle by the chain.[5]

Formula (5) shows that in the low-frequency region the radiation intensity does not depend on the frequency ω.

For $\xi \ll 1$ and $\theta \gg \theta_c$, formula (5) gives the result of the Ter-Mikaelyan theory[8] of coherent radiation by particles in a crystal. As $\theta \to 0$, Eq. (5) implies that the intensity of radiation decreases. Thus, in the low-frequency region the radiation intensity (5) is maximum at $\theta \sim \theta_c$. For angles in this region the radiation by superbarrier particles is more intense than the radiation by particles in an amorphous medium by about a factor of $R/4d\theta_c \ln(183Z^{-1/3})$.

5. For $\theta \ll \theta_c$ a certain number of the particles incident on the crystal are captured into the channeling regime. The spectral distribution of radiation by the superbarrier and channeled particles. The latter should lead to a sharp maximum in the radiation

spectrum (against the background of radiation by superbarrier particles) in the frequency region $\omega \sim 4\pi\gamma^2/T$, where T is the average period of oscillation of the channeled particles. If, when a beam is passed through a crystal, the number of unchanneled particles is large, the radiation will be governed by the superbarrier particles at all frequencies.

In order to detect the features predicted in this letter for the radiation by superbarrier particles in the low- and high-frequency regions, it will be necessary to conduct experiments under conditions where channeling is absent: for example, to examine the radiation from particles in a crystal for $\theta \sim \theta_c$.

The author wishes to thank A. I. Akhiezer for discussion of difficult questions and I. I. Miroshnichenko for discussion of the results of the experiment reported in Ref. 1.

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