Scaling theory of localization and the concept of minimum metallic conductance ($d = 3$)

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The scaling theory of charge-carrier localization in disordered semiconductors[1] (see also Ref. 2), can lead to rather radical consequences. Although the critical comments[3,4] appear to be very serious, the logical relationship between them is not completely clear: objections of an analytical character are based on a consideration of scattering with spin flipping, whereas the Anderson Hamiltonian, which contains no random magnetic field, was used for a numerical calculation.

In this paper we focus attention on the fact that the concept of minimum metallic conductance in a three-dimensional problem can be correlated with the basic idea of scaling theory[1,2] if one of the not so obvious assumptions of this theory is rejected. Specifically, it was assumed in Refs. 1 and 2 that the dimensionless conductance g defined by the equation

$$g = \frac{2\hbar}{e^2} \sigma L^{d-2}, \tag{1}$$

is everywhere a continuous and differentiable function of the scaling parameter L and hence of the dimensionless energy $\epsilon = (E - E_c)/E_c$ (E_c is the mobility threshold and σ is the normal conductance of the material). This assumption rejects the idea of a nonzero, mimimum metallic conductance σ_m. In fact, if $\sigma_m \neq 0$, then the $\sigma(\epsilon)$ function must have a first-order discontinuity point for $\epsilon = 0$. If (and only if) the scale parameter is assumed to be a continuous function of ϵ, then the $g(\epsilon)$ function [or $g(L)$] must also have a singularity at $\epsilon = 0$. We assume that the $g(L)$ function is sufficiently smooth at all the remaining points. Thus, the basic equation of the scaling theory remains valid everywhere, except at the singular point, (we use the same symbols as those in Refs. 1 and 2):

$$d \ln g / d \ln L = \beta(g). \tag{2}$$

The asymptotic form of the $\beta(g)$ function for $g \gg 1$ and $g \ll 1$ were investigated in Refs. 1 and 2. In particular, for $g \gg 1$ we have $\beta(g) \to d - 2$. This asymptotic form remains valid in our problem if $g > g_c = (2h/e^2)\sigma_m L^{d-2}$. Focusing attention only on the three-dimensional problem, we can disregard the correction term of the order of g^{-1} that is possible in principle (but placed in doubt in Refs. 3 and 4). For $g = g_c$, $\beta(g)$ the function must have a singularity; thus, Eq. (2) must be written separately for $g > g_c$ and $g < g_c$, and as $g \to g_c$ (from above or from below) the left-hand side of Eq. (2) should be considered the derivative from the right or from the left.

We consider the region $\epsilon > 0 (g > g_c)$. We obtain, according to (2), for g close to g_c

$$L \cong L_0 \exp\left\{ \frac{1}{g_c} \int_{g_0}^{g} \frac{dg}{\beta(g)} \right\}, \tag{3}$$

where $g_0 = g_c + a\epsilon^m$ is some initial point $(a > 0, m > 0)$.

Assuming that

$$\beta(g) = \nu (g - g_c)^n. \quad 1 > n > 0, \tag{4}$$

we find

$$L \sim \exp\left\{ -\frac{(a\,\epsilon^m)^{1-n}}{\nu g_c (1-n)} \right\}.$$

It follows from this that for $d = 3$

$$\sigma \sim \exp\left\{ \frac{(a\,\epsilon^m)^{1-n}}{\nu g_c (1-n)} \right\}. \tag{5}$$

This expression remains different from zero as $\epsilon \to 0$. Thus, we can see that the concept of a minimum metallic conductance can be retained in scaling theory, if certain systematic properties are not linked to the $\beta(g)$ function *a priori*. In other words the assumption that the $\beta(g)$ function has a certain analytical structure as $g \to g_c$ is equivalent to the assumption that a minimum metallic conductance is absent or present in a three-dimensional system. This means that additional independent information is needed for an unambiguous solution of the σ_m problem.

[1]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 693 (1979).
[2]E. Abrahams and T. V. Ramakrishnan, J. Non-Cryst. Solids **35**, 15 (1980).
[3]P. Lee, Phys. Rev. Lett. **42**, 1492 (1979).
[4]P. Lee, J. Non-Cryst. Solids **35**, 21 (1980).