New type of quantum size effects in metals produced as a result of multichannel specular reflection (MSR) of electrons from the boundaries of a sample

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(Submitted 14 July 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 5, 363–366 (5 September 1980)

It is shown that under MSR conditions the electron kinetics in thin metallic plates are qualitatively different from the quasiclassical case and have essentially a quantum-interference nature.

PACS numbers: 61.80.Mk

1. Recent experimental studies[1] have shown that the reflection of conduction electrons from metal-vacuum boundaries is often close to specular. Because of the anisotropy and band structure of the electron dispersion law $\epsilon_m(\mathbf{p})$ ($\mathbf{p}$ is the quasi-momentum and m is the band number), the multichannel specular reflection (MSR) is a typical situation for metals. In the presence of MSR an electron incident on a metal boundary which is parallel to one of the crystallographic planes [the (x,y) plane below] makes a transition generally from a given state $|\mathbf{p},m\rangle$ to a superposition of several (N)

states $|\mathbf{p}',m'\rangle$ with the same energy $E=\epsilon_{m'}(\mathbf{p}')=\epsilon_m(\mathbf{p})$ and momentum projection $\mathbf{p}_\parallel=(p_x,p_y)$, but with an opposite sign of the transverse velocity $v_z=\partial\epsilon_{m'}(p')/\partial p'_z$ (see the example in Fig. 1, where $N=2$). This process is described by the scheme

$$|\alpha_+\rangle \rightarrow \sum_{\alpha'=1}^{N} s_{\alpha\alpha'}(\mathbf{p}_\parallel,E)\,|\alpha'_-\rangle,\quad \alpha=1,\ldots,N, \tag{1}$$

where the subscript α indexes the sets of quantum numbers (p,m) with given E, $\mathbf{p}_\parallel$, and the sign v_z, and $+$ and $-$ denote the states with velocities directed toward and away from the boundary, respectively. $s_{\alpha\alpha'}$ are the smoothly dependent (on the scale of the Fermi energy ϵ_F and Fermi momentum p_F probability amplitudes of the transitions $|\alpha_+\rangle\rightarrow|\alpha'_-\rangle$ which form the unitary N-channel s matrix of the MSR.

2. At first glance, it is natural to consider the electron transitions due to MSR as random jumps of a classical electron with probabilities $W_{\alpha\alpha'}=|s_{\alpha\alpha'}|^2$. In this paper we show that for a sufficiently good boundary specularity and $d\lesssim l_0$ (d is the thickness of the metallic plate and l_0 is the electron mean free path) this stochastic description is not applicable, and the MSR dynamics of the electron have a quantum-interference nature that results in many unusual kinetic properties of electrons in thin plates.

First, we shall examine the motion of a quasi-classical wave packet that is slightly smeared with respect to momentum and coordinate in the scale of p_F and d, respectively. The packet, after undergoing MSR at the plate boundary, is split into N packets in accordance with Eq. (1). Each of the resulting packets is split again due to MSR at the plate boundaries, etc. As a result, the number of electron packets increase exponentially with time. The amplitude of each packet at time t can be uniquely determined by specifying one of the "Brownian" paths (they are called "j paths" below), which a classical particle traverses during the time t, while undergoing random transitions $\alpha_+\rightarrow\alpha'_-$ at the plate boundary due to MSR. Each such transition contributes a factor $s_{\alpha\alpha'}$ to the packet amplitude; in addition, the amplitude contains the quasi-classical phase factor $\exp\{iS_j/\hbar\}$, where S_j is the classical action increment along a given j path, which depends on E and on the initial $\mathbf{p}_\parallel$. It is important that for $t\gg d/v_F$ ($v_F\sim\epsilon_F/p_F$) there are exponentially many packets with identical $S_j(E,\mathbf{p}_\parallel)$, which interfere with each other as a result of superimposition, i.e., their amplitudes are added. The total amplitude of the interfering packets is the Feynman integral along the trajectories which are degenerate because of the quasi-classical nature of the motion within the plate in a summation along the j paths.

As can be shown, this interference pattern corresponds to the energy levels of the size quantization $E_n(\mathbf{p}_\parallel)$ (n is the level number)—the zeros in the dispersion equation

$$\mathrm{Det}\,\|\delta_{\alpha\alpha'}-\sum_{\alpha''} s_{\alpha\alpha''}s^*_{\alpha'\alpha''}\exp\{i(\phi^{(-)}_{\alpha''}-\phi^{(+)}_{\alpha'})\}\|=0, \tag{2}$$

where the quasi-classical phases are $\phi^{(\pm)}_\alpha=dp^{(\pm)}_{z,\alpha}/\hbar$ and $p^{(\pm)}_{z,\alpha}-z$ is the z component of $\mathbf{p}$ in the $|\alpha_\pm\rangle$states. (In writing Eq. (2), we assumed that the plate boundaries are identical.) Since the phase differences $|\phi^{(-)}_{\alpha''}-\phi^{(+)}_{\alpha'}|\sim d/\lambda\gg 1$ (the wavelength

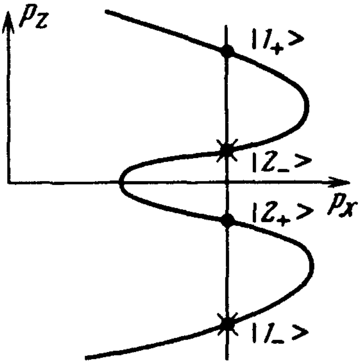

FIG. 1. A cut of the Fermi surface by the p_z,p_x plane is shown; the points and crosses denote the states with opposite directions of the transverse velocity.

$\lambda \sim \lambda/p_F$) and their derivatives with respect to E and $\mathbf{p}_\parallel$ are incommensurable, the terms $E_n(\mathbf{p}_\parallel)$, together with the matrix elements of the physical quantities, oscillate rapidly and randomly with respect to $\mathbf{p}_\parallel$ with a small quantum variation scale $\sim \hbar/d << p_F$. This is the fundamental property that distinguishes the MSR spectrum from the locally equidistant, quasi-classical spectrum corresponding to ordinary (one-channel) specular reflection. This gives rise to the anomalous sensitivity of the MSR kinetics of thin plates to any weak, external field which are capable of changing $\mathbf{p}_\parallel$ by an amount $\delta p_\tau \gtrsim \hbar/d$ during the relaxation time $\tau = l_0/v_F$.

3. These conclusions indicate that the conductivity of the thin plates deviates from Ohm's law at very small electric field intensities $\mathscr{E} \sim \bar{\mathscr{E}} = \hbar/ed\tau$. For $\mathscr{E} \ll \bar{\mathscr{E}}$, when the normal linear theory is applicable, the conductivity σ_{xx} is strongly dependent on the amplitudes $s_{\alpha\alpha'}$. Thus, for a configuration that is symmetrical with respect to the reflections $p_z \to -p_z$ the calculations give

$$\sigma_{xx} = \frac{4e^2\tau}{(2\pi\hbar)^3}\int d\mathbf{p}_\parallel \left[\frac{|v_x^{(1)}|^2}{|v_z^{(1)}|^2} + \frac{|v_x^{(2)}|^2}{|v_z^{(2)}|^2} - \frac{\sqrt{1-W}\,(|v_x^{(1)}| - |v_x^{(2)}|)^2}{\sqrt{v_+^2 - Wv_-^2}}\right]\Bigg|_{E=\epsilon_F}, \tag{3}$$

where $W = |s_{11}|^2, v_\pm = |v_z^{(1)}| \pm |v_z^{(2)}|$, and $|v_{x,z}^{(\alpha)}| = |\partial\epsilon/\partial p_{x,z}|$ for the $|\alpha_\pm\rangle$ states. In the limit $\mathscr{E} \gg \bar{\mathscr{E}}$ the variation $\mathbf{p}_\parallel$ in an electric field during the time τ results in a unique averaging of the longitudinal velocity $\partial E_n/\partial p_x$ over the fast, random "flickers" of the MSR spectrum. In this case the current density J appears in the universal, linear asymptotic from $J = \sigma_{xx}^{(\infty)}\mathscr{E}$, where the coefficient $\sigma_{xx}^{(\infty)}$ is smaller than the σ_{xx} of linear theory and does not depend on the elements of the s matrices:

$$\sigma_{xx}^{(\infty)} = \frac{e^2\tau}{(2\pi\hbar)^3}\int d\mathbf{p}_\parallel [\Sigma(v_x/|v_z|)]^2/\Sigma|v_z|^{-1}. \tag{4}$$

The summation in Eq. (4) extends to all $|\alpha_\pm\rangle$ states with given $\mathbf{p}_\parallel$ and $E = \epsilon_F$. Equation (4) corresponds to an "ergodic" (uniform) distribution of an electron over all the states in Eq. (4) under the Σ sign. In the intermediate $\mathscr{E} \sim \bar{\mathscr{E}}$ region it is theoretically possible for a decreasing region to exist on the volt-ampere characteristic.

The transition from "linear" conductivity σ_{xx} to ergodic conductivity $= \sigma_{xx}^{(\infty)} < \sigma_{xx}$ occurs in any external field for which $\delta p_\tau \sim \hbar/d$. For a magnetic field **H** with $H_z \sim |\mathbf{H}|$ the quantity $\delta p_\tau \sim (eH/c)l_0$. Therefore, for example, for $d \sim 10^{-4}$ cm and $l_0 \sim 10^{-3}$ cm the influence of the magnetic field on the conductivity is significant at fields $H \sim 10^{-1}$ cgs units. Nonlinear anomalies of this nature are also possible in high-frequency fields. Another interesting aspect of the MSR is the complication of the quantum size oscillations of the kinetic coefficients whose periods d, as shown on the basis of Eq. (2), are all possible integer combinations of the differences

$$(1/p_{z,\alpha}^{(\pm)} - 1/p_{z,\alpha'}^{(\pm)}),\ (1/p_{z,\alpha}^{(\pm)} - 1/p_{z,\alpha'}^{(\pm)}).$$

To observe the interference effects discussed above and to fulfill the inequality $d \lesssim l_0$, the characteristic dimension L of the flat sections of the sample boundary must be $\gtrsim d$. Otherwise, large (and random) phase differences $S_j/\hbar$, which transform a quantum electron to a classical particle performing Brownian motion in the plate, appear in the interfering packets. This stochastization (see Sec. 2) corresponds to disintegration of the MSR spectrum. For $d \lesssim L$ the broadening of the levels of the MSR spectrum is $\sim Q\hbar v_F/d$ (Q is the diffusion coefficient) and, consequently, it is sufficient that $Q \ll 1$ in order to observe the effects due to the MSR spectrum.

It must be noted that the MSR leads to a situation similar to that discussed above in a magnetic field parallel to the boundary. The Larmor radius r_H must be $\lesssim L$ for an interference pattern to appear in this case.

The results of a detailed analysis of the effects mentioned here and a description of the calculation procedure with a quasi-random MSR spectrum will be submitted for publication soon.

The authors thank I. M. Lifshitz, N. E. Alekseevskiĭ, B. I. Verkin, Yu. P. Gaĭdukov, and Yu. F. Komnik for valuable discussions.

[1]A. F. Andreev, Usp. Fiz. Nauk **105**, 113 (1971) [Sov. Phys. Usp. **14**, 609 (1971)].

Translated by Eugene R. Heath
Edited by S. J. Amoretty