Model of the magnetic transition at the interface of solid and liquid He_3

S. V. Iordanskiĭ and A. V. Smirnov
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

(Submitted 17 July 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 6, 422–425 (20 September 1980)

By using a phenomenological model it is shown that a ferromagnetic transition, which is due to the existence of a layer whose density is between those of the solid and liquid phases, can occur at the interface of solid and liquid He_3.

PACS numbers: 75.30.Kz, 67.50.Dg, 67.80.Jd

Experiments performed for liquid He_3 in graphoid[1] showed that a surface contribution to the magnetic susceptibility corresponds to ferromagnetism. This fact was explained[2] by describing He_3 near the graphite surface as a solid with a small number of vacancies. It was shown more recently[3] that vacancies in a triangular, two-dimensional lattice produce ferromagnetism. Calculations were made in Ref. 4 for the Bethe-Salpeter equation for a Fermi liquid in a potential well near a solid wall. This method also showed that ferromagnetism can exist.

We hope to explain in this paper these two approaches in a phenomenological model, since the parameters and the existence of surface magnetism are very difficult to determine from first principles. A phenomenological treatment was carried out in Ref. 5; however, our model is somewhat different in its physical scope.

A relatively strong exchange in liquid He_3 produces a Fermi energy $\epsilon_F \sim 2.5$ K. A small exchange in solid He_3 produces a transition temperature $T_N \sim 1$ mK.

The decrease in the role of exchange is due to an increase of the correlation energy because of the solid He_3 cores; this can be observed by using the infinite-repulsion Hubbard model.[6]

For a small filling of the band, the energy of the ground state consists of the energy of an ideal Fermi gas plus a correction for the finite amplitude of the particle scattering from each other. As the density increases, the role of the second term increases, and at some critical density of the particles it becomes more advantageous to have the same spin with single filling of each band level, since in this case there are not scattering corrections because of the Pauli principle. There is not exchange (for infinite repulsion) in a completely filled band, but there is a spin degeneracy. A real, solid He_3 has a weak exchange mechanism even in the absence of vacancies,[7] which gives rise to an antiferromagnetic transition.

Although the problem of zero vacancies in He_3 has been discussed in the literature,[8] there is no evidence of their existence.

This situation, however, can change in the nonequilibrium case. According to theoretical predictions and experimental data,[9] there are reversible oscillations of the interface of liquid and solid He_3 (on the melting curve); this is interpreted as the absence of kinks on the solid He_3 surface and the existence of a transition layer with a gradual density variation. In a simplified approach we can assume that the liquid is a solid with many vacancies, which decrease and eventually vanish in the solid phase. A nonequilibrium number of vacancies (from the viewpoint of the bulk properties) occur in the intermediate layer, where the free-energy terms, which depend on the density gradient, stabilize such a structure.

To realize a ferromagnetic transition in the intermediate layer, the exchange in the direction of the density gradient must be smaller than the exchange in the perpendicular plane. Simple considerations suggest that such a decrease must occur, since resonance tunneling occurs in the plane, but not in the transverse direction. We assume that there is a ferromagnetic Fermi liquid in layer 1 and a normal Fermi liquid in layer 2. If the interaction between the layers is described by the usual tunnel Hamiltonian, assuming that the Fermi bottom filling is shifted in layer 2 by an amount $\delta U > 0$ relative to layer 1, then we can easily obtain the interaction energy of the layers by using perturbation theory

$$E_{int} \sim \frac{t^2}{\epsilon_F + \delta U}(n_0 - n),$$

where $n_0 - n$ is the density of vacancies and t is the probability that an atom jumps into the nearest equilibrium state. Since the exchange in the layer is characterized by the same constant t, the exchange efficiency decreases in the ratio $\kappa = t/(\epsilon_F + \delta U)$.

It is important however, that a ferromagnetic transition can occur without a large decrease of the interlayer exchange. To show this, we examine the phenomenological model with a thermodynamic potential per unit area

$$\Omega = \int \left\{ \frac{1}{2} a\left(\frac{dn}{dx}\right)^2 + \frac{1}{2} \beta\left(\frac{dS}{dx}\right)^2 + \omega(n,S) \right\} dx,$$

where the first two terms characterize an energy increase due to the density gradient n and spin S, and ω is the local value of the thermodynamic potential density $\omega = f - \mu n$, where f is the free-energy density and μ is the chamical potential. This value, which describes the magnetic phase transition in the layer, corresponds to two thermodynamic equilibrium conditions: Fermi liquid and a solid without vacancies. The value β is associated with the interlayer exchange and hence contains the factor κ, in contrast to the exchange terms in ω. As is customary in the Landau theory, we shall assume that

$$\omega(n, S) = \omega_0(n) + \tfrac{1}{2} \chi^{-1}(n) S^2 + \gamma S^4 .$$

The susceptibility is $\chi(n) < 0$ in the intermediate layer, which is determined by the density interval n_{c_2} corresponds to the decay of ferromagnetism because the entropy or direct exchange becomes stronger at a small concentration of vacancies. The susceptibility is positive outside this layer, which we shall assume has a thickness of ν atomic layers of size a. The value $\gamma > 0$, since saturation must occur in S.

It is easy to show that the larger value of Ω for $S \neq 0$ in the intermediate layer is equivalent to a negative eigenvalue λ_0 in the Schrödinger equation

$$-\frac{d}{dx}\beta\frac{dS_{\chi_0}}{dx} + \chi^{-1}(x)S_{\chi_0} = \chi_0 S_{\chi_0}.$$

The remaining eigenvalues generally do not play an important role, since they differ from λ_0 by an integer.

If the values χ and β are piecewise constant and assume values of $\chi_s < 0$ and β_s in the intermediate layer of thickness νa, then we can show that the negative eigenvalue must appear when the inequality $\pi < \nu a(\beta_s \chi_s)^{-1/2} \sim \nu \kappa^{-1/2}$ is satisfied, which is not a rigorous constraint on the smallness of κ even for $\nu = 1$.

As the temperature increases, the interval in which the susceptibility is negative decreases, since progressively higher vacancy concentrations are required to compensate for the entropy loss due to spin ordering. This, will occur first in the solid-phase, where the vacancy concentration is the lowest. A direct exchange in the solid phase should not result in qualitative changes. A competition between the ferromagnetic vacancy exchange and the antiferromagnetic direct exchange near the point with a density n_{c_2} reduces the effective magnetic interaction, so that this point under all circumstances will shift toward the liquid as the temperature increases. At some critical temperature the eigenvalue $\chi_0(T)$ vanishes, and we have a second-order transition within the context of the Landau theory.

Thus, according to our model, the ferromagnetic phase transition is an intrinsic property of the interface of solid and liquid He_3, regardless of whether we are on the melting curve or whether the solid helium is formed as a result of the action of a substrate.

[1]A. J. Ahonen *et al.*, Phys. Rev. Lett. **41**, 494 (1978).
[2]R. A. Guyer, Phys. Rev. Lett. **39**, 1091 (1977).
[3]S. V. Iordanskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 361 (1979) [JETP Lett. **29**, 325 (1979)]; S. V. Iordanskiĭ and A. V. Smirnov, Zh. Eksp. Teor. Fiz. **79**, No. 11 (1980) [Sov. Phys. JETP **52**, No. 5 (1980) (in press)]
[4]M. T. Beal-Monod and S. Doniach, J. Low Temp. Phys. **28**, 175 (1977).
[5]P. Kumar, Preprint Low Temp. Lab. Helsinki University of Technology, 1979,
[6]Y. Nagaoka, Phys. Rev. **147**, 392 (1966).
[7]T. C. Prewitt and J. M. Goodkind, Phys. Rev. Lett. **39**, 1283 (1977).
[8]A. F. Andreev and I. M. Lifshits, Zh. Eksp. Teor. Fiz. **56**, 2058 (1969) [Sov. Phys. JETP **29**, 1107 (1969)].
[9]A.F. Andreev and A. Ya. Parshin, Zh. Eksp. Teor. Fiz. **75**, 1511 (1978) [Sov. Phys. JETP **48**, 763 (1978)].

Translated by Eugene R. Heath
Edited by S. J. Amoretty