Instantons in a quark plasma. A two-loop correction.

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A two-loop correction for the influence of an instanton immersed in a cold ($T \to 0$) plasma consisting of light quarks is calculated.

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As is known, corrections generated by instantons that are nonanalytic in the coupling-constant (i.e., by the field configurations that have a topological charge) play an important role in the calculation of physical quantities in quantum chromodynamics.[1] Since the quantum fluctuations reduce the influence of an instanton as the radius increases,[2] the large, strongly overlapping instantons will dominate.

It was shown that the field of the heavy, real quarks can suppress the pseudoparticles[3] and that the instanton gas within the hadrons is diluted.

An instanton immersed in a quark plasma is of interest in the study of the interaction with light quarks. Initially it appeared that since the plasma shields the long-wave fluctuations, the size of instantons would be limited by the Debye radius.[4]

We have investigated the dependence of the action S of an instanton on the chemical potential μ of a plasma. In first approximation we can assume that the quarks are massless and solve the problem by using exact Green's functions in the instanton field.[5,6]

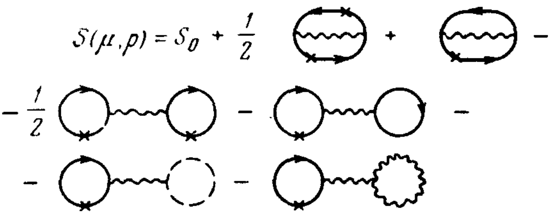

FIG. 1. Two-loop diagrams for the action of an instanton in a quark plasma. The cross represents the lines of real quarks. The wavy lines correspond to the gauge field and the dashed line corresponds to the "ghost" field.

A correction for the instanton effect is missing in the leading order of perturbation theory, i.e., the instanton does not interact with the plasma (Ref. 6).[1]

The results of two-loop calculations are of more interest. The $S(\mu,\rho)$ dependence (for $T=0$) is expressed by the equation $[\alpha=\alpha(\mu)=g^2(\mu)/4\pi]$ in the main logarithmic approximation.

$$S = \frac{2\pi}{\alpha} - \frac{1}{3}(11N_c - 2N_f)\ln \mu\rho$$
$$- \frac{N_c^2-1}{N_c} N_f \frac{\alpha}{4\pi}(\mu\rho)^2 \ln \frac{4\pi}{\alpha\mu\rho}\sqrt{\frac{3N_c}{N_f(N_c^2-1)}} - \frac{N_f^2}{2}\frac{\alpha}{4\pi}(\mu\rho)^4 \ln \frac{4\pi\sqrt{6}}{\alpha(\mu\rho)^2 N_f}.$$

Here ρ is the radius of an instanton, and N_c and N_f are the number of colors and types of quarks. The first two terms represent the ordinary $S(\rho)$ dependence in a vacuum. An addition can be obtained by calculating the diagrams in Fig. 1.

The $S(\mu\rho)$ dependence is graphically represented in Fig. 2. The action of an

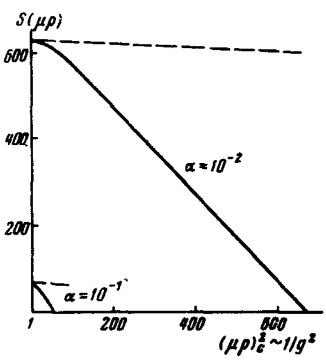

FIG. 2. $S(\mu,\rho)$ dependence for different values of $\alpha(\mu)$. The dashed line represents the result of a one-loop calculation.

instanton can be decreased by immersing it in a medium. It also sharply decreases as its size increases. The $S(\mu,\rho)$ curves cross the X axis of the Debye shielding radius at $\rho = \rho_c \sim \rho_D$.

The density of quarks near an instanton increases and the field is strongly distorted inside a pseudoparticle (for $r < r_{\text{in}} \sim \rho^2/\rho_D$) and outside of it $(r > r_{\text{out}} \sim 1/(\sqrt{g})\rho\sqrt{\rho_D/\rho})$. The perturbation theory in an instanton field can be used if $r_{\text{in}} < \rho < r_{\text{out}}$, i.e., if $\rho < \rho_D$. The value of ρ_c corresponds to the limits of applicability of the perturbation theory, i.e., Eq. (1) is self-consistent.

Instantons that are larger than the Debye radius strongly interact with the medium and do not exist independently. To describe them, we should solve the equations of motion of the field with allowance for matter.

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[1)]The results[6] can be easily generalized to the case $T \neq 0$.

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