# Production of massless particles by a conformally flat, gravitational field

A. D. Dolgov
*Institute of Theoretical and Experimental Physics*

(Submitted 23 October 1980)

Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 11, 673–677 (5 December 1980)

It is shown that the gluon anomaly in the trace of the energy-momentum tensor is responsible for the production of massless vector bosons in an isotropic gravitational field.

PACS numbers: 04.40. + c, 12.25. + e

The impossibility of the production of massless particles by an isotropic, gravitational field was first asserted in Refs. 1 and 2 and later repeatedly confirmed in the literature (see, for example, review article[3]). The argument, in short, is as follows. It is assumed that the massless particles are described by a conformally invariant theory. On the other hand, the space-time metric in the case in question can be converted by a conformal transformation to a Minkowski-space metric in which there is no production of particles. The assertion is based on this fact.[1] The explicit calculations[1-3,5-7] for the noninteracting particles showed that their production probability in the isotropic spaces vanishes in the massless limit. This result turned out to be valid[8] for the scalar particles with the $\lambda\phi^4$ interaction. The production of particles may generally be allowed, however, if the interaction between the particles is taken into account.

The goal of this paper is to show that the massless particles can nonetheless be produced by an isotropic, gravitational field. The fact is that the quantum corrections associated with the loop diagrams generally break down the conformal invariance of the theory, despite the conformal invariance of the original Lagrangian of the elementary particles. This occurs because of the need for introduction of heavy, control masses in the calculation of the diverging diagrams. Because of this, in particular, a well-known anomaly occurs in the trace of the energy-momentum tensor of particles (in the flat space):

$$T_{\mu\mu} = \sum_j \left( m_j^2 \, |\phi_j|^2 + \widetilde{m}_j \, \bar{\psi}\psi \right) + \frac{\alpha}{8\pi} \beta G^a_{\mu\nu} G^a_{\mu\nu}, \tag{1}$$

where $\phi_j$ are the operators of quantum boson fields with a mass $m_j$, $\psi_j$ are those of fermion fields, $G^a_{\mu\nu}$ is a stress tensor of vector gauge fields, and $\alpha$ is the gauge coupling constant. The $\beta$ coefficient depends on the theory. Specifically, $\beta = (11/3)N - (2/3)N_f$ for a gauge theory based on the $SU(N)$ group with $N_f$ fermion generations.

Equation (1) is the starting point of our discussion. It is important that $T_{\mu\mu}$ does not vanish at $m = 0$.

We analyze a weak, gravitational field when the metric has the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the metric tensor in the Minkowski space and deviations from the flat metric are small ($|h_{\mu\nu}| \ll 1$). The amplitude of a pair of particles with momenta $k_1$ and $k_2$, which are produced by the gravitational field, is

$$A(k_1 k_2) = \int d^4 x \, h_{\mu\nu} \langle k_1, k_2 | T_{\mu\nu} | 0 \rangle . \tag{3}$$

We analyze the isotropic case when the metric can be selected in the form

$$h_{00} = 0, \; h_{0i} = 0, \; h_{ik} = h(t)\, \delta_{ik} . \tag{4}$$

We assume that $h(t \to \pm\infty) = 0$.

The total momentum of the produced particles must be equal to zero for a homogeneous space. Therefore, the transversality condition, $\partial_\mu T_{\mu\nu} = 0$, leads to the disappearance of the matrix elements $T_{\mu 0}$. Because of this, the particle production amplitude is proportional to the $T_{\mu\mu}$ trace for the metric of (4). If the last anomalous term in Eq. (1) is not taken into account, then we can see that the production probability of massive bosons is proportional to the fourth power of the mass, consistent with the known results. The last term in Eq. (1) shows, however, that because of conformal anomaly, the field (4) can produce pairs of massless vector bosons.

The energy density of the produced vector particles can be easily calculated

$$\rho = \frac{1}{V} \int \frac{d^3 k_1 d^3 k_2}{4 E_1 E_2 (2\pi)^6} (E_1 + E_2) | A (k_1, k_2)|^2 , \tag{5}$$

where $A$ is determined by Eq. (3), $V$ is a normalized, three-dimensional volume, and $E_i$ and $k_i$ are the energy and momentum of the produced particles.

After a small calculation, we obtain (for $m_j = 0$):

$$\rho_0 = \frac{a^2 \beta^2 K}{2^7 \pi^2} \int_0^\infty d\omega \omega^5 | \tilde{h}(\omega) |^2 , \tag{6}$$

where $\tilde{h}(\omega) = (1/2\pi) \int dt\, e^{i\omega t} h(t)$, $K$ is the number of particles, and $K = N^2 - 1$ for $SU(N)$.

In spite of the fact that the production of gauge bosons by a gravitational field occurs in the high order of the interaction constant $\alpha$, in contrast with the case of massive particles whose production is possible in zeroth order in $\alpha$, the latter nonetheless are produced much more weakly, even if the process occurs near the gravitational singularity. The ratio of the energy density (6) to the energy density of massive bosons produced by the same gravitational field in this case is

$$\rho_o / \rho_m \approx \frac{a^2 \beta^2}{2^6 \pi^2} (m_P / m)^4 , \tag{7}$$

where $mp = 10^{19}$ GeV is the Planck mass. This ratio is of the order of $10^{13} \alpha^2 \beta^2 \gg 1$ even for superheavy bosons with $m \approx 10^{15}$ GeV, which were analyzed in the "grand-unification" schemes. In particular, the particle production near the singularity must occur rather efficiently in the Friedmann universe.

Strictly speaking, the equations cited here are incorrect near the gravitational singularity, since the constraint of the low-order gravitational interaction may be invalid there. However, a strong production of massless vector bosons must remain qualitatively valid.

We note that in describing the interaction of scalar particles with the gravitational field, we use either the standard Klein-Gordon equation in the curvilinear coordinates

$$( \partial^\mu \partial_\mu + m^2 ) \phi = 0 \tag{8a}$$

or the conformal invariant modification of this equation

$$( \partial^\mu \partial_\mu + m^2 - \frac{1}{6} R ) \phi = 0, \tag{8b}$$

where $R$ is the scalar curvature.

The first case corresponds to the operator of the energy-momentum tensor of the form

$$T^{(0)}_{\mu\nu} = \partial_\mu \phi \, \partial_0 \phi - \frac{1}{2} \eta_{\mu\nu} [(\partial_\alpha \phi)^2 - m^2 \phi^2]. \quad (9a)$$

The second case corresponds to the addition to $T^{o}_{\mu\nu}$ of the total derivative

$$\tilde{T}_{\mu\nu} = T^{(0)}_{\mu\nu} + \frac{1}{6} (\eta_{\mu\nu} \partial_\alpha \partial_\alpha - \partial_\mu \partial_\nu) \phi^2. \quad (9b)$$

We can easily see that

$$T^{(0)}_{\mu\mu} = 2m^2 \phi^2 - (\partial_\mu \phi)^2, \quad \tilde{T}_{\mu\mu} = m^2 \phi^2 \quad (10)$$

and the gravitational vertex for the isotropic metric vanishes in the massless limit in the second case but does not vanish in the first case. This is consistent with the known conclusions concerning the possibility of particle production in the theory with Eq. (8a) and the absence of production for Eq. (8b).

If the $\lambda\phi^4$ interaction is taken into account, then the equations of motion and the energy-momentum tensor will, of course, be modified, but the condition (10) will not change. This accounts for the lack of production of interacting particles, which was mentioned in Ref. 8.

We also note that the results of Ref. 7 can be easily reproduced in the weak-field limit for the nonisotropic metric by using Eq. (5).

I thank A. I. Vaĭnshteĭn, V. I. Zakharov, Ya. B. Zel'dovich, and A. A. Starobinskiĭ for useful discussions.

1)We note that since the wave equation for gravitons is not conformally invariant, the gravitons can be produced in a conformally flat universe.[4]

---

1. K. A. Bronnikov and É. A. Tagirov, Preprint P2-4151, J.I.N.R., 1968.
2. L. Parker, Phys. Rev. Lett. 21, 562 (1968); Phys. Rev. 183, 1057 (1969).
3. L. Parker, in "Asymptotic Structure of Spacetime," edited by F. Esposito and L. Witten, Plenum, New York, 1977.
4. L. P. Grishchuk, Zh. Eksp. Teor. Fiz. 67, 825 (1974) [Sov. Phys. JETP 40, 409 (1974)].
5. A. A. Grib and S. G. Mamaev, Yad. Fiz. 10, 1276 (1979); 14, 800 (1971) [Sov. J. Nucl. Phys. 10, 722 (1979); 14, 450 (1971)].
6. L. Parker, Phys. Rev. D3, 346 (1977)).
7. Ya. B. Zel'dovich and A. A. Starobinskiĭ, Zh. Eksp. Teor. Fiz. 61, 2161 (1971) [Sov. Phys. JETP 34, 1159 (1972)].
8. L. Parker, Phys. Rev. D7, 976 (1973).

Translated by S. J. Amoretty
Edited by Robert T. Beyer