Spontaneous compactification of subspace due to interaction of the Einstein fields with the gauge fields

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(Submitted 30 October 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 11, 681–684 (5 December 1980)

The mechanism for compactification of spaces due to interaction of the Einstein fields with the gauge fields, which is different from the mechanism previously analyzed by Cremmer and Scherk, is determined.

PACS numbers: 04.90. + e

The approximate method of compactification of additional space-time measurements is now used extensively in constructing dual models with internal symmetries and Lagrangians for $O(N)$ broadened supergravitation.[1–3] It was shown by Cremmer and Scherk[1–3] and later by Luciani in a more general way that compactification of additional measurements under certain conditions may be due to the original Lagrangian, i.e., it can have a spontaneous nature. In this paper we examine the mechanism of spontaneous compactification of subspaces, which is different from the mechanism examined in Ref. 2.

Assuming that the vacuum state of the fields in the subspace subject to compactification is determined by zero values of all the fields except the Einstein fields and the gauge fields, we examine the Lagrangian corresponding to them

$$L = \alpha \sqrt{g} R + \beta \sqrt{g}\, F^{\alpha\beta}_{lm} F^{\alpha\beta\, lm}. \qquad (1)$$

The indices $l, m = 1, 2, \ldots n$ and $\alpha, \beta = 1, 2, \ldots n$ pertain to the space and intrinsic variables, respectively. $(F^{\alpha\beta})_{lm} = -(F^{\beta\alpha})_{lm}$ are the strengths of the $O(n)$ gauge group.

The ratio α/β can be larger or smaller than zero, depending on whether the examined subspace is spacelike or timelike with respect to the signature of the metric tensor of the total space in which the real space and time are included with its usual signature.

The field equations have the usual form,

$$a\left(R_{lm} - \frac{1}{2} g_{lm} R\right) = - T_{lm} , \tag{2}$$

$$T_{lm} = \beta \left(F_{l\ s}^{\ \alpha\beta} F_{m}^{\alpha\beta\ s} - \frac{1}{4} g_{lm} F_{ps}^{\alpha\beta} = F^{\alpha\beta\, ps}\right) \tag{3}$$

and

$$(D_l F_{m}^{\ l})^{\alpha\beta} = 0, \tag{4}$$

where D_l represents a covariant derivative that contains both a metric connectivity defined by Christoffel symbols Γ^l_{mn} and the gauge fields.

To find a nontrivial vacuum solution, the Riemannian space, which is determined by Eq. (2), must be a space of constant, positive curvature, i.e.,

$$R_{lmnp} = K (g_{mn} g_{lp} - g_{ln} g_{mp}) , \quad K > 0 \tag{5}$$

and the gauge fields must satisfy the additional condition

$$(D_l F_{mn})^{\alpha\beta} = 0, \tag{6}$$

which identically satisfies Eqs. (4).

If we take Eq. (5) into account, then Eq. (6) will have the following solution for the strengths

$$(F_{lm})^{\alpha\beta} = - K (\chi^{\alpha}_{l} \chi^{\beta}_{m} - \chi^{\alpha}_{m} \chi^{\beta}_{l}), \tag{7}$$

and for the gauge fields

$$(A_m)^{\alpha\beta} = - \chi^{\alpha p} \Gamma^{s}_{mp} \chi^{\beta}_{s} + (\partial_m \chi^{\alpha}_{s}) \chi^{s\beta}, \tag{8}$$

where χ^{α}_{m} are continuous functions that satisfy the orthogonality conditions

$$\chi^{\alpha}_{l} \chi^{\alpha}_{m} = g_{lm}, \qquad g^{lm} \chi^{\alpha}_{l} \chi^{\beta}_{m} = \delta_{\alpha\beta} . \tag{9}$$

The relation (8) establishes a connection between the gauge connectivity and the metric connectivity.

For the specified functions χ^{α}_{m}, which corresponds to the selection of a specific gauge, the expressions (7) and (8) are invariants of the global $O(n+1)$ group that coincides with the metric-invariance group.

Substituting the strengths (7) in Eq. (2) and taking Eq. (3) into account, we obtain the following equation:

$$a (n - 2) K = \beta (n - 4) K^2 , \tag{10}$$

which determines the dependence of the Gaussian curvature on the ratio α/β. Equation (10) has two solutions for n unequal to two and four $-K=0$, which corresponds to the trivial case of the flat space, and

$$K = \frac{\alpha(n-2)}{\beta(n-4)}, \quad (11)$$

for a space of constant curvature.

Thus, as a consequence of the nonlinear equation (10), which is in a certain sense analogous to the equation for spontaneous symmetry breaking by Higgs bosons, the Lagrangian (1) contains a spontaneous transition in which the flat space is transformed into spheres.

It follows from the relation (11) that $\alpha/\beta<0$ for $n=3$ and $\alpha/\beta>0$ for $n>4$. As a result, compactification can occur only if the space is timelike for $n=3$ and spacelike for $n>4$.

In conclusion, we note that the investigation performed above can be extended, without substantial changes, to compactification of spaces for which

$$R_{lm;np,r} = 0 \quad (12)$$

i.e., to the general case of symmetrical spaces.[5] The gauge group for the fields $F^{\alpha\beta}_{mn}$ in this case must coincide with the holonomy group of symmetrical space.

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Translated by S. J. Amoretty
Edited by Robert T. Beyer