How can fractional charge of quasiparticles be observed?

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(Submitted 24 September 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 9, 372–374 (10 November 1985)

The minimum charge of the quasiparticles formed in an incompressible quantum fluid is calculated. The strong dependence of the energy gap on the magnitude of this charge provides a way to test the theoretical conclusions.

The fractional quantum Hall effect discovered by Tsui *et al.*[1] has become the subject of active research. Laughlin[2] has suggested quasiparticles with a fractional charge in an effort to explain this effect. Laughlin suggested that the electrons at the first Landau level in a strong magnetic field may, at certain concentrations, form an incompressible quantum fluid. These concentrations are

$$\nu \equiv \frac{N}{2S} = \frac{1}{m}, \tag{1}$$

where N is the number of electrons, $2S$ is the number of flux quanta through the area occupied by electrons, and m is an odd integer. Laughlin showed that if the number of flux quanta differs by unity from that given by (1) at the given value of N, the extra or deficient flux quantum is bound to a local change in the electron density. This change in electron density may be interpreted as a quasiparticle floating in an incompressible homogeneous quantum fluid. Making use of the formal analogy with a one-component plasma, Laughlin found that the modulus of the effective charge of the quasiparticles is

$$|e^*| = |e|/m, \tag{2}$$

where e is the charge of an electron.

Since the magnitude of the quasiparticle charge plays an important role in a calculation of the transverse conductivity[3] σ_{xy}, it is interesting to consider another derivation of (2), based on gauge invariance.[4] The results derived by Arovas *et al.*[4] were found only near filling factors $\nu = 1/m$. In the present letter we calculate the minimum charge of quasiparticles near other rational values $\nu = p/q$.

We first consider the particular case $\nu = 1/m$, and we offer a simplified derivation of the result of Ref. 4. The wave function proposed by Laughlin[2] for a quasihole at $\nu = 1/m$ is

$$\psi = \prod_{i=1}^{N} (z_i - \zeta)\, \psi_L (z_1, \ldots, z_N), \tag{3}$$

where

$$\psi_L = \prod_{\alpha < \beta}^{N} (z_\alpha - z_\beta)^m \exp\Big[- \sum_{l=1}^{N} |z_l|^2/4\Big]. \tag{4}$$

Here z represents the complex coordinates of the electrons, and ζ is the coordinate of the quasihole. It was shown in Refs. 5 and 6 that (3) is the exact wave function of the ground state when the Coulomb interaction between electrons is replaced by a short-range interaction.

Let us examine a large closed contour Γ within which there are n electrons. The passage of a quasihole around this contour changes the phase of wave function (3) by an amount $(-2\pi n)$. On the other hand, it follows from gauge invariance that this phase change is equal to $(e^*/\hbar c)\phi$, where ϕ is the magnetic flux through the area bounded by contour Γ. If $\nu = 1/m$, there are m flux quanta $\phi_0 = 2\pi\hbar c/e$ per electron. Equating the phase changes found by the two methods, we find

$$-2\pi n = \frac{e^*}{\hbar c}\; nm\phi_0 = 2\pi nm\,\frac{e^*}{e}\;. \tag{5}$$

Hence we find result (2).

We do not actually have to know the exact form of the wave function of the quasiparticles in order to calculate their charge.

Let us assume that, for some rational filling factor $\nu = p/q$, the wave function is nondegenerate. We change the number of flux quanta by ± 1. If an additional quantum is bound to a local charge in the electron density, it is a straightforward matter to find the charge of this local formation. As an electron traverses a contour around a localized quantum, the phase of the wave function changes by $\pm 2\pi$. Accordingly, when a localized quantum moves around an electron, the phase changes by $\mp 2\pi$. Repeating the arguments above, we find that the charge bound to a localized flux quantum is $\mp e\nu$.

If ν is not $1/m$, but, say, $\nu = p/(mp \pm 1)$, where m is odd, and p is an integer, it is a more complicated matter to find the quasiparticles of minimum charge. According to Haldane[7] and Halperin,[8] such filling factors arise from Laughlin states with $\nu = 1/m$ if the quasiholes or quasielectrons themselves form an incompressible Laughlin fluid with a density of $1/p$. New quasiparticles of minimum charge are found by simultaneously increasing the number of flux quanta and the number of quasielectrons of charge e/m by unity. The minimum possible charge is

$$e^*_{m,p} = \pm\,\frac{e}{m(mp \pm 1)}\;. \tag{6}$$

In particular, the charge of the quasielectrons near a filling factor $\nu = 2/5$ is $e/15$, while near $\nu = 2/7$ we find $e^* = e/21$.

A similar analysis yields $e^* = e/35$ at $\nu = 3/7$.

The energy gap in the spectrum of excitations of an electron fluid at $\nu = p/q$ is given in order of magnitude by

$$\epsilon \sim \frac{(e^*)^2}{\kappa l^*_H}\,, \tag{7}$$

where κ is the dielectric constant, and l^*_H is the magnetic length, given by

$$l^*_H = \sqrt{\frac{\hbar c}{|e^*|H}}\;. \tag{8}$$

We note that we have $\epsilon \sim (e^*)^{5/2}$. For approximately equal values of the filling factor ν, the values of the gap ϵ may be greatly different. For example, we have

$$\frac{\epsilon(\nu=2/5)}{\epsilon(\nu=1/3)} \sim \left(\frac{1}{5}\right)^{5/2} \approx 0{,}02 \ . \qquad (9)$$

By measuring the gap in the spectrum we can thus verify whether the charges of the quasiparticles at $\nu = p/(mp \pm 1)$ do in fact obey relation (6).

The magnitude of ϵ also determines the maximum temperature at which a step exists on the Hall characteristic.

[1]D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[2]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[3]V. L. Pokrovskiĭ and A. L. Talapov, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 68 (1985) [JETP Lett. **42**, 80 (1985)].
[4]D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).
[5]V. L. Pokrovskiĭ and A. L. Talapov, J. Phys. C **18**, L691 (1985).
[6]S. A. Trugman and S. Kivelson, Phys. Rev. B **31**, 5280 (1985).
[7]F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).
[8]B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).

Translated by Dave Parsons