Spontaneous breaking of axial symmetry in v-vortices in superfluid ^{3}He-B

G. E. Volovik and M. M. Salomaa

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow; Low-Temperature Laboratory, Technical University of Helsinki, Finland

(Submitted 16 October 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 10, 421–424 (25 November 1985)

A new, axially asymmetric state of a broken-parity vortex in ^{3}He-B has been found. At low pressures, this state has an energy lower than that of an axisymmetric state. Only a first-order phase transition can occur between these two states. The properties of a rotating liquid, which are associated with the breaking of axial symmetry in a vortex, are discussed.

Since the discovery of a first-order phase transition within the core of a quantized singular vortex in the quasi-isotropic B phase of superfluid ^{3}He (Ref. 1), involving an abrupt change in the magnetic moment concentrated in the core,[2] there has been active theoretical research on the structure of the core. Near the temperature (T_c) of the superfluid transition, that axisymmetric vortex which has the lowest energy is the so-called v-vortex,[3] in whose core spatial parity P is broken but the combined parity PTU_2 is conserved; here T is time reversal, and U_2 is the revolution of the vortex line. Because of parity breaking, the v-vortex has a spontaneous electric polarization and a spontaneous spin current, which are concentrated in the core and which are directed along the axis of the vortex. Superfluidity is not disrupted in the core of a v-vortex: The core consists primarily of A phase, whose orbital angular momentum $\mathbf{l}$ is directed along the axis, and also of β phase, with ferromagnetically ordered spins of Cooper pairs.

In the A phase, however, the energetically preferred orientation of the orbital angular momentum $\mathbf{l}$ is in the plane of the flow. This circumstance has the consequence, in particular, that quantized vortices in ^{3}He-A, either singular or nonsingular, do not have axial symmetry.[4] We are thus led to ask whether axial symmetry is broken in a v-vortex. To answer this question, we have studied the stability of an axially symmetric state of a v-vortex in the Ginzburg-Landau region with respect to perturbations that break axial symmetry.

The order parameter in superfluid ^{3}He, whose Cooper pairs are in the $S = 1$, $L = 1$ state, contains nine amplitudes $a_{\mu\nu}$, which correspond to states with different projections (μ and ν) of the spin S and of the orbital angular momentum L. In a vortex, $a_{\mu\nu}$ depends on the distance (r) from the vortex axis and on the azimuthal angle φ and can be written in the following general form:

$$a_{\mu\nu}(\mathbf{r}) = \sum_Q C^{(Q)}_{\mu\nu}(r)e^{i(Q+1-\mu-\nu)\varphi} . \quad (1)$$

The term with $Q=0$ describes the axisymmetric state of a vortex in ^{3}He-B with a single quantum of circulation; the other terms are perturbations that break axial symmetry. The perturbations with different values of $|Q|$ do not mix in the linear approximation.

The most important perturbations are those with $|Q|=1$ and $|Q|=2$. An instability with respect to perturbations with $|Q|=1$ gives rise, through the nonlinearity, to the appearance of all other harmonics; i.e., the symmetry is broken with respect to all rotations around the vortex axis. In this case, a special direction arises in the plane perpendicular to the axis of the vortex; this direction is characterized by the unit vector **b**, which changes sign under time reversal ($T\mathbf{b}=-\mathbf{b}$). This event corresponds precisely to a deviation of the orbital angular momentum **l** from the direction of the vortex axis. An instability with $|Q|=2$ generates only even harmonics of Q and corresponds to a conservation of the symmetry C_2: a rotation of π around the vortex axis. In this case the vector **b** becomes a two-sided director, and the orbital angular momentum does not deviate from the axis.

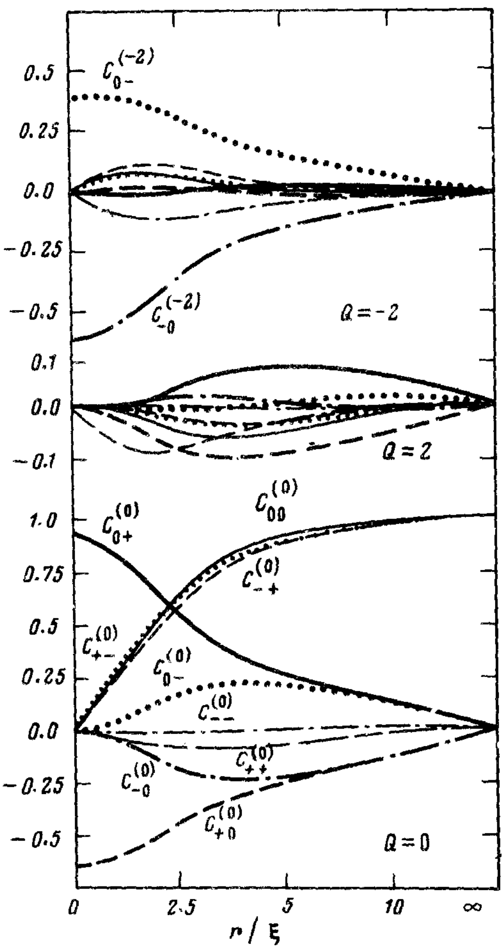

FIG. 1. Components of the order parameter in a v-vortex with broken axial symmetry versus the distance from the vortex axis (ξ is the coherence length). The harmonics $C^{(0)}_{\mu\nu}$ correspond to the axisymmetric part of the order parameter. The notation for the harmonics $C^{(+2)}_{\mu\nu}$ and $C^{(-2)}_{\mu\nu}$ is the same as for $C^{(0)}_{\mu\nu}$. At the vortex axis, the components C_{0+}, C_{0-}, C_{+0}, C_{-0} are nonzero. The approximate equality $C_{+0}\cong C_{-0}$ holds, indicating that the so-called axially planar phase, first described by Mermin and Ster,[5] exists at the axis.

We have found that in the approximation of so-called weak coupling, which corresponds to low pressures, the axisymmetric state of a v-vortex is linearly stable with respect to small perturbations with both $|Q| = 1$ and $|Q| = 2$. However, large-amplitude perturbations with $|Q| = 2$ (but not with $|Q| = 1$) lower the energy of the v-vortex. Figure 1 shows the result of a minimization of the Ginzburg-Landau energy functional in the class of functions with $Q = 0, +2, -2$. At low pressures, the axial symmetry is thus broken in the vortex, so that we are left with only the discrete symmetry elements C_2 (for even Q) and PTU_2 ($C_{\mu\nu} = C^*_{\mu\nu}$). Whether this symmetry is also broken will be shown by future research.

The absence of an instability in the small means that the axial symmetry can be broken only by a first-order phase transition. Accordingly, if it turns out that the axial symmetry is not broken for vortices at high pressures, we would have an explanation of the observed first-order phase transition. This possibility is indicated by the results of a numerical analysis carried out by Thuneberg.[1] The breaking of axial symmetry in a v-vortex, while retaining all the former properties of the v-vortex which are associated with parity breaking, gives rise to some new properties. First, an additional Goldstone mode arises: oscillations of the vector **b** which propagate along the vortex axis. Second, there is a modification of the interaction of vortices with the volume order parameter, which is specified by the rotation matrix $R_{\alpha i}(\mathbf{n}, \theta)$ that couples the spin and orbital subsystems in the B phase. The rotation angle θ is fixed by the spin-orbit (dipole) interaction $\cos\theta_0 = -1/4$, and the rotation axis **n** is oriented by the magnetic field **H** and the vortices. The orientation of **n** with respect to **H** is measured in NMR experiments. The total orientational energy, found after an average is taken over vortices in a volume element, has the general form

$$F = a\left\{ -(\mathbf{nH})^2 + \frac{2}{5}H^2\left[\lambda_{11}(\mathbf{h}\hat{\vec{\Omega}})^2 + \lambda_{12}(\mathbf{h}\hat{\vec{\Omega}})(\mathbf{hb}) + \lambda_{22}(\mathbf{hb})^2\right] + \frac{4}{5}H\left[\kappa_1(\mathbf{h}\hat{\vec{\Omega}}) + \kappa_2(\mathbf{hb})\right]\right\}, \quad h_i = R_{\alpha i}H_\alpha/H. \tag{2}$$

Here $\hat{\vec{\Omega}}$ is a unit vector along the rotation axis (i.e., along the axis of the vortices), so that we have $\mathbf{b}\perp\vec{\Omega}$. The first term in (2) describes the interaction of **n** with the magnetic field due to the slight magnetic anisotropy in the volume. The second term describes the interaction (quadratic in the field) of the order parameter with vortices which arises from the pronounced magnetic anisotropy near a core. We are assuming that because of this interaction, the vectors **b** of the individual vortices should be oriented identically. In the case of C_2 symmetry, we have $\lambda_{12} = 0$, and in the axisymmetric case we also have $\lambda_{22} = 0$. The third term describes the gyromagnetic energy associated with the existence of a magnetic moment $M_\alpha = -(4/5)R_{\alpha i}(\kappa_1\vec{\Omega}_i + \kappa_2 b_i)$, which is concentrated in the vortex cores. We have $\kappa_2 = 0$ if there is C_2 symmetry.

Third, solitons can exist at a vortex. The soliton is a part of a vortex line in which the vector **b** changes by 2π (or by π, if there is C_2 symmetry). Away from a soliton, the vector **b** is fixed by the interaction with the order parameter in the volume.

All five of the parameters λ and κ can be found from NMR experiments. In particular, whether C_2 symmetry is broken can be determined by applying a field $\mathbf{H}\|\vec{\hat{\Omega}}$. In this case the equilibrium angle (β) between **n** and **H** is nonzero only if the parameter λ_{12} is nonzero:

$$\sin^2 \beta = \frac{\lambda_{12}^2}{10\,(1-\lambda_{11}+\lambda_{22})^2} \quad . \tag{3}$$

This equality is found by minimizing (2) under the constraints $\lambda_{12} \ll 1$, $\lambda_{11}-\lambda_{22}<1$. New measurements and also a reexamination of the old data on the basis of (2) should provide the key to the identification of the phase transition in a vortex, since it will thus become possible to determine whether there is a breaking of axial symmetry, and the nature of the breaking, in each of the two vortex states that are observed.

This paper is a result of joint studies carried out within the framework of the ROTA Soviet-Finnish Scientific Project.

[1)]E. Thuneberg, private communication.

[1]O. T. Ikkala, G. E. Volovik, P. Yu. Khakonen, Yu. M. Bun'kov, S. T. Islander, and G. A. Kharadze, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 338 (1982) [JETP Lett. **35**, 416 (1982)].
[2]P. J. Hakonen, M. Krusius, M. M. Salomaa, J. T. Simola, Yu. M. Bunkov, V. P. Mineev, and G. E. Volovik, Phys. Rev. Lett. **51**, 1362 (1983).
[3]M. M. Salomaa and G. E. Volovik, Phys. Rev. B **31**, 203 (1985).
[4]H. K. Seppälä, P. J. Hakonen, M. Krusius, T. Ohmi, M. M. Salomaa, J. T. Simola, and G. E. Volovik, Phys. Rev. Lett. **52**, 1802 (1984).
[5]N. D. Mermin and G. Ster, in: Novosti fundamental'noĭ fiziki (News in Fundamental Physics), Moscow, 1977, No. 7, p. 121.

Translated by Dave Parsons