Dynamics of vortex pairs in a two-dimensional magnetic material

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A parametric resonance in an external rf field can be used to detect topological excitations in planar two-dimensional ($2d$) magnetic materials.

Berezinskiĭ[1] has shown that vortex excitations in the low-temperature phase of a planar $2d$ magnetic material combine into bound states—vortex pairs—which are in a thermal equilibrium. At a critical temperature T_c, calculated by Kosterlitz and Thouless,[2] the molecules dissociate and a vortex plasma forms. The statistical data on vortex pairs have been studied theoretically in detail. Unfortunately, experimental verification of the theory has encountered some difficulties: a weak anisotropy and the distortion of the two-dimensional picture by the three-dimensional interactions in layered systems.

In the present letter we study the dynamics of vortex pairs in $2d$ ferromagnets and, in particular, the parametric resonance in an rf field. The interactions mentioned above are unimportant in the study of the dynamics.

The equations of motion for single moving vortices were derived by Huber[3] and Nikiforov and Sonin.[4] They studied a weak, easy-plane anisotropy, a physically interesting case in which the spins at the vortex core emerge from the plane. A vortex is therefore characterized not only by the circulation q but also by the number σ, which has a value of $+1$ or -1 and which indicates the spin direction in the vortex core. Instead of the motion of a single vortex, it is more appropriate to consider the motion of a system of vortices whose total circulation $Q = \Sigma_i q_i$ is equal to zero. The equation of motion for such a system of vortices at low velocities $\mathbf{v}_i$ is

$$\mathbf{G}_i \times \mathbf{v}_i + D\mathbf{v}_i = \sum_{k \neq i} \mathbf{F}_{ik} + \mathbf{F}_i . \quad (1)$$

Let us explain the notation. The gyration vector

$$\mathbf{G} = 2\pi q \sigma \hat{z} \quad (2)$$

is directed along the normal to the plane; the unit vector in this direction is denoted by $\hat{z}$; D is the dissipation coefficient

$$D = \alpha_0 \ln R/l . \quad (3)$$

Here α_0 is a dimensionless constant, R is the spacing between the vortices, and l is the anisotropy length which is equal to the size of the vortex core. The force at which the

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i-th vortex interacts with the k-th vortex is determined by the relative position of the vortex centers which are characterized by the vectors $\mathbf{R}_i$ and $\mathbf{R}_k$:

$$\mathbf{F}_{ik} = \frac{\mathbf{R}_i - \mathbf{R}_k}{|\mathbf{R}_i - \mathbf{R}_k|^2} 2\pi g J q_i q_k \; ; \quad g = s_0/\hbar , \tag{4}$$

where g is the gyromagnetic factor, s_0 is the area of a unit cell, and J is the exchange constant (saturation magnetization is assumed to be unity). The external force $\mathbf{F}_i$ is determined by the external magnetic fields. Since we are considering only the uniform fields $\mathbf{H}(t)$, we can write

$$\mathbf{F}_i = - g\mathbf{H}\, \partial \mathbf{M} / \partial \mathbf{R}_i . \tag{5}$$

Let us consider an isolated vortex pair with $Q = 0$. Disregarding the dissipation in the absence of external forces, we can write the equation $\sigma_1 \mathbf{v}_1 - \sigma_2 \mathbf{v}_2 = 0$, from which we see that there are two types of vortex pairs. If $\sigma_1 = \sigma_2$, a vortex pair will move collectively in the direction perpendicular to the line connecting their centers at a velocity $v = \gamma/2R$, where $\gamma = 2gJ$. The second type of vortex pairs ($\sigma_1 = -\sigma_2$) rotate around a fixed common center at an angular velocity $\omega = \gamma/R^2$.

Dissipation causes a vortex pair to shrink in size (R) in proportion to

$$R^2 = R_0^2 - 2DG\gamma\, t(G^2 + D^2)^{-1} . \tag{6}$$

The energy and total magnetic moment of a vortex pair depend solely on R if the discrete nature of the lattice is disregarded. This dependence is obvious in the case of exchange interaction. It also applies, however, in the case of a uniaxial anisotropy in a plane, since the energy and total magnetic moment are invariant with respect to the rotation of the plane without a spin flip. The symmetry is disrupted by only the dipole interaction, which accounts for a slight dependence of the energy and total magnetic moment of a vortex pair on the angle of rotation of the vortex pair (ϕ) with respect to the preferential direction of magnetization, assumed to be the x axis. The dependence of the magnetic moment on R and ϕ, for example, can be written as follows:

$$M_x = M_0(R) + M_1(R)\cos 2\phi + \dots \; ; \qquad M_y = M_1(R)\sin 2\phi + \dots \; . \tag{7}$$

The value of M_1 can be calculated by multiplying the transverse component of the true magnetic field induced by all the spins of the system at a given point on the plane by the magnetic-moment gradient, calculated at the same point for a single-vortex static solution, and then integrating over the entire plane. As a result, we find

$$M_1 = \frac{\mu_B^2}{\gamma\hbar} \frac{s}{s_0} R \ln\frac{R}{l} , \tag{8}$$

where $s \sim J/\beta$ is the area of a vortex core.

Because of the asymmetry of the magnetic moment, a vortex pair can be excited parametrically by an alternating external field $\mathbf{H}(t)$. For definiteness, we assume that $\mathbf{H}$ is directed along the y axis: $H_y = H_0\cos\omega_0 t$. Using (2), (4), (5), and (7), we can reduce equation of motion (1) to the form

$$\begin{aligned} -\dot{y} + \bar{D}\dot{x} &= -\gamma x/R^2 - hM_1 y^3/R^4 , \\ \dot{x} + \bar{D}\dot{y} &= -\gamma y/R^2 - hM_1 x^3/R^4 \end{aligned} \tag{9}$$

where $h = h_0\cos\omega_0 t$ $(h_0 = g\mu_B H_0/\pi s_0)$ is the reduced magnetic field, and $\bar{D} = D/2\pi$. Since x^3 and y^3 contain the first and third harmonics, it is reasonable to expect a resonance to occur at the external-field frequency ω_0, which is approximately equal to $2\omega(R)$. In the neighborhood of the resonance, we will use for Eqs. (9) a method for averaging over the fast variable, $\Phi = \int\omega(R)dt$. As the slow variables we use R and α, which are related to the x and y coordinates by

$$x = R\cos(\Phi + \alpha), \qquad y = R\sin(\Phi + \alpha).$$

In a first approximation for the small parameter $\delta = M_1 h_0/\gamma$, Eqs. (9) reduce to the following equation for a slow, single variable $\psi = \int(2\omega(R) - \omega_0)dt + 2\alpha$:

$$A\ddot{\psi} + \cos\psi - B = 0, \tag{10}$$

where $A = 4/(\omega_0^2\delta)$, and $B = 2\bar{D}/\delta$.

Equation (10) is consistent with a mechanical analogy: This is an equation of motion for a particle with a mass A in an external field with a periodic component and a constant component. The potential of this field

$$V(\psi) = -B\psi + \sin\psi \tag{11}$$

is plotted in Fig. 1. The bound states in potential (11), which appear when $B < 1$, ccrrespond to quasi-steady regimes of the rotating vortex pairs. The trapping of vortex pairs begins at the cutoff field, $H_c = \mu_B^{-1} j\beta s_0(\mu_B^2/s_0^{3/2})^{-1}(s_0\omega/s\omega_{\max})^{1/2}\bar{D}/\ln^R/l$, where $\omega_{\max} = \gamma/l^2$. The spacing ΔR between the vortices of a vortex pair trapped by the external field is

$$\Delta R = \omega_0^{-1} R\sqrt{2\Delta V A^{-1}}, \tag{12}$$

where ΔV is the potential-barrier height (Fig. 1). Near the cutoff field, we have

$$\Delta V = \frac{4\sqrt{2}}{3}(1 - H_c/H)^{3/2}.$$

The resonant absorption is determined by the density of the vortex pairs that are

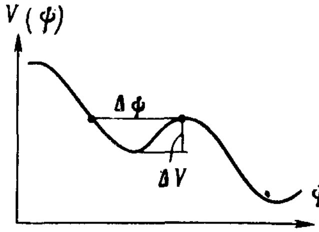

FIG. 1.

trapped by the external field, $N_{tr} = 1/2\, W(R)R\Delta R\Delta\psi$. Here $W(R) = W_0 R^{-4T_c/T}$ is the Boltzmann weight of a vortex pair, the spacing ΔR between the vortices of a vortex pair is given by Eq. (12), and $\Delta\psi$ is the interval of the phases corresponding to the captured states (Fig. 1). Near the cutoff field, we have $\Delta\psi \sim (1 - H_c/H)^{1/2}$. Each trapped vortex pair absorbs a power

$$\delta Q = \frac{1}{g} D v^2 .$$

The total power of the *rf* field absorbed per unit area is

$$Q(\omega_0) = N_{tr}\,\delta Q \sim \left(\frac{J^2}{\hbar s_0}\right)\left(\frac{\omega_0}{2\omega_{max}}\right)^{2T_c/T} \bar{D}^{3/2}\left(1 - \frac{H_c}{H}\right)^{5/4}\left(\frac{H}{H_c}\right)^{-1/2} .$$

It should be pointed out that no parametric spin-wave resonance will occur if the external field is oriented relative to the magnetizing field in the manner considered above. In principle, a resonance can occur not only at a doubled frequency but also at a quadrupled frequency [see Eqs. (9)].

Finally, let us numerically estimate the quantities that characterize a parametric resonance. A useful object for such an estimate is a layered magnetic material $(C_2H_5NH_3)_2CuCl_4$, whose static properties were studied in detail by de Jongh *et al.*[5] The parameters that describe this resonance are the resonant frequency

$$\omega_0 < \omega_{max} = \frac{J}{\hbar}\,\frac{s_0}{s} \quad (\nu_0 = \omega_0/2\pi \sim 10^9\,\mathrm{Hz})$$

the cutoff field

$$H_c \sim 10^3 \bar{D}(s_0/s)^{1/2}\ \mathrm{Oe}$$

and the power absorbed per cm^3

$$Q \sim 10^{12}\bar{D}^{3/2}\left(1 - \frac{H_c}{H}\right)^{5/4}(\omega_0/2\omega_{max})^{2T_c/T}(H_0/H)^{1/2}\ \mathrm{W}.$$

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