Defects and an unusual superconductivity

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If the superconducting gap in a pure material has zeros on an entire line along the Fermi surface, the state density is finite at $\epsilon = 0$, beginning with the lowest defect concentrations. The splitting of the superconducting transition by impurities, $U_{1-x}Th_xBe_{13}$ is discussed.

Arguments in favor of a nontrivial nature of the superconductivity in heavy-fermion systems are based on the power-law behavior of several measured properties at low temperatures (the specific heat,[1] the ultrasonic absorption coefficient,[2] and NMR line widths,[3] among others). All the superconducting classes that are possible for these systems were listed in Refs. 4 and 5. It turns out that in the triplet class ($S = 1$) the superconducting gap can vanish only at isolated points on the Fermi surface, while in the singlet case ($S = 0$) it can vanish on entire lines along the Fermi surface. Depending on the particular features of the gap, the specific heat (for example) would vary in proportion to T^3 and T^2, respectively ($T \ll T_c$).

Let us examine the stability of these results with respect to impurities. We first assume $S = 0$; then the order parameter[4] is a scalar, $\psi(\mathbf{k})$. To determine the state density for excitations, we use a generalization of the expression in Ref. 6 for the entropy:

$$S_e = -\frac{1}{(2\pi)^3 T^2}\int_0^\infty \frac{\epsilon\, d\epsilon}{\mathrm{ch}^2(\epsilon/2T)}\frac{dS_{\mathbf{k}}}{v(\mathbf{k})}\,\mathrm{Im}\sqrt{|\tilde{\psi}(\mathbf{k},\epsilon)|^2 - \tilde{\epsilon}^2(\mathbf{k},\epsilon)}\,. \tag{1}$$

The functions $\tilde{\epsilon}(\mathbf{k},\epsilon)$ and $\tilde{\psi}(\mathbf{k},\epsilon)$ are the result of an analytic continuation from the upper thermodynamic frequency axis ($\epsilon = i\epsilon_n$) of the solution of the equations

$$\tilde{\epsilon}_n = i\epsilon_n + \frac{1}{2}\left\langle \tau^{-1}(\mathbf{k},\mathbf{k}')\frac{\tilde{\epsilon}}{\sqrt{|\tilde{\psi}|^2 - \tilde{\epsilon}^2}}\right\rangle_{\mathbf{k}'}, \tag{2}$$

$$\tilde{\psi}(\mathbf{k}, i\epsilon_n) = \psi(\mathbf{k}) + \frac{1}{2}\left\langle \tau^{-1}(\mathbf{k},\mathbf{k}')\frac{\tilde{\psi}}{\sqrt{|\tilde{\psi}|^2 - \tilde{\epsilon}^2}}\right\rangle_{\mathbf{k}'}.$$

Here $\langle\ldots\rangle_{\mathbf{k}'} = \int\ldots dS_{\mathbf{k}'}/\int dS_{\mathbf{k}}$.

Equations (2) determine the self-energy parts of the Green's functions averaged over impurities. For a uniform impurity distribution, the kernel $\tau^{-1}(\mathbf{k},\mathbf{k}')$ has the initial crystal symmetry. The symmetry of $\tilde{\psi}(\mathbf{k},\epsilon)$ is thus the same as $\psi(\mathbf{k})$; in particular, $\tilde{\psi}(\mathbf{k},\epsilon)$ has zeros at the same locus of points.

For simplicity, we assume an isotropic impurity [$(\tau^{-1}(\mathbf{k},\mathbf{k}') \equiv \tau^{-1}$]. The second of

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Eqs. (2) gives us $\tilde{\psi} = \psi$, while the first becomes

$$\tilde{\epsilon} = i\epsilon_n + \frac{1\tilde{\epsilon}}{2\tau} \langle (|\psi|^2 - \tilde{\epsilon}^2)^{-1/2} \rangle .$$

If $\psi(\mathbf{k})$ vanishes on lines, the integral here has a logarithmic singularity,

$$\tilde{\epsilon}\left(1 - \frac{1}{2\tau\Delta_0} \ln \frac{2\Delta_0}{-i\tilde{\epsilon}}\right) = \epsilon \tag{3}$$

$[T \to 0, |\epsilon|, |\tilde{\epsilon}| \ll \Delta_0$, where Δ_0 is the scale dimension of $|\psi(\mathbf{k})|]$. The value $\epsilon = 0$ corresponds to

$$\tilde{\epsilon}_0 = i2\Delta_0 \exp(-2\tau\Delta_0). \tag{4}$$

Expanding (3) in the neighborhood of (4), we find from (1) a finite state density in a narrow region $\epsilon \simeq 0$:

$$\nu_S / \nu_N = 4\tau^2\Delta_0^2 \exp(-2\tau\Delta_0). \tag{5}$$

The general form of the kernel $\hat{\tau}^{-1}(\mathbf{k},\mathbf{k}')$ is

$$\hat{\tau}^{-1}(\mathbf{k},\mathbf{k}') = \sum_{i,\lambda} \tau_i^{-1} \hat{\theta}_\lambda^{i}(\mathbf{k}) \hat{\theta}^{i\lambda}(\mathbf{k}'), \tag{6}$$

where the operator symbol incorporates the spin variables, the index i specifies all the eigenvalues τ_i^{-1}, $\hat{\theta}^{i,\lambda}$ are the corresponding eigenfunctions, which always belong to one of five representations of the cubic group, and λ specifies the basis functions of the representation if it is degenerate. The sign of τ_i^{-1} is, in general, arbitrary, except for the principal value, $\tau_0^{-1} > 0$, which belongs to the identity representation. We now see that (4) and (5) are also valid in the general case, since the corrections to $\tilde{\psi}(\mathbf{k},\epsilon)$ do not contain logarithmic terms. Finally, if the zeros of the gap correspond to isolated points on the Fermi surface, a perturbation theory will be valid at low concentrations. A "gap-free superconductivity" is possible only at a finite defect concentration, $\tau\Delta_0 \sim 1$.

In the case of a nontrivial superconductivity, impurities will lower T_c. It was found in Ref. 7, however, that the superconducting transition in $U_{1-x}Th_xBe_{13}$ also splits in two. An explanation for this effect[8] was proposed on the basis of the suggestion that the superconductivity in UBe_{13} is anisotropic and is characterized by a vector whose direction is pinned by defects in a random way. The second transition (at $T_{c2} < T_{c1}$) was identified with an orientational transition. The estimates in Ref. 8, however, contain some parameters which were literally small. In this connection, it was mentioned in Ref. 8 that in the fourth-degree terms in the Ginzburg-Landau functional

$$\beta_1 |\vec{\eta}|^4 + \beta_2 |\vec{\eta}^2|^2 + \beta_3(|\vec{\eta}_x|^4 + |\vec{\eta}_y|^4 + |\vec{\eta}_z|^4) \tag{7}$$

($\vec{\eta}_i$ are the coefficients in the expansion of the order parameter in the basis functions of the three-dimensional representations of the O group) the relations between the coefficients β_i depend on the impurities. The derivations of the coefficients β_i in the weak-coupling model, which is completely analogous to that of Ref. 9, shows that all the β_i vary over a rather broad range, depending on the assumptions regarding the anisotropy of the basis functions and the signs of τ_i^{-1} [several terms corresponding to different representations must be retained in (6)]. Of these results we will mention two: As the impurity concentration x is increased, the superconducting transition may become a first-order transition; i.e., (7) may lose its positive definite form. Second, in the weak-coupling model and for $x=0$ we have $\beta_2 = \frac{1}{2}\beta_1 > 0$ for $S=0$, and, according to Ref. 5, the superconducting phase near T_c corresponds to a symmetry $D_4(E)$ or $D_3(E)$. In other words, it has a magnetic moment. For triplet pairing, $S=1$, we have $\beta_2 < 0$ (the latter result was also derived in Ref. 10). Correspondingly, there are classes[5] $D_3 \times R$, $D_3(C_3)\times R$, $D_4(C_4)\times R$, $D_4^{(2)}(D_2)\times R$, in which the order parameter is real.

Returning to the possibility of describing the splitting of the transition in $U_{1-x}Th_xBe_{13}$ by this approach (we are assuming that the second transition is also a superconducting transition), we are confronted by the circumstances that these classes exhaust all the extrema of the Ginzburg-Landau functional[5] (7). Continuous transitions between them are not possible within the framework of a single representation (this also applies to two-dimensional representations). Measurements of NMR line widths[3] indicate that the second transition is continuous.

The splitting of the temperature of the superconducting transition can be understood easily by assuming that there is a preferred orientation of the impurities and that this orientation lowers the symmetry of the cube. That this is indeed the case in the UBe_{13} system was brought to our attention by N. E. Alekseevskiĭ, whom the authors thank. We also thank G. E. Volovik and D. E. Khmel'nitskiĭ for many useful discussions.

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