Stability of a spherical EHD

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The stability of the electron-hole drop (EHD) relative to its change of shape is examined in terms of the hydrodynamic approach, taking into account the surface tension and elastic strain.

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As can be seen from the experiment, the drops of an electron-hole liquid (EHL) in a semiconductor irradiated by a laser usually are not larger than 10^{-3} cm. Larger drops are formed in the regions of specially produced elastic strain. Although a number of hypotheses, such as the "phonon wind," have been advanced to explain this fact,[1] there is yet no clearly formulated theory. In this note we investigate the stability of spherical drops as one of possible mechanisms which limits their size in the hydrodynamic approximation.

We assume that the friction of exciton gas and of the EHL against the lattice (because of the phonons or impurities) is sufficiently large. Therefore, disregarding the nonlinear terms and the compressibility of EHL, we obtain an equation for the interior of the drop:

$$\rho_L \frac{\partial \mathbf{v}_L}{\partial t} + \frac{\rho_L \mathbf{v}_L}{\tau_L} = -\nabla p_L - n_L \nabla U, \qquad \operatorname{div} \mathbf{v}_L = -\frac{1}{\tau_0}. \tag{1}$$

Here $\mathbf{v}$ is the velocity, ρ is the density, p is the pressure, n is the density of particles in the EHL, U is the potential energy of the elastic strain, which is assumed to be isotropic, τ_0 is the recombination time, and τ_L is the momentum loss time in the EHL. Analogously, in the exciton pair:

$$\rho_P \frac{\partial \mathbf{v}_P}{\partial t} + \frac{\rho_P \mathbf{v}_P}{\tau_P} = -\nabla p_P - \frac{p_P}{T} \nabla U,$$

$$\frac{\partial \rho_P}{\partial t} + \operatorname{div} \rho_P \mathbf{v}_P = 0, \qquad p_P = \frac{\rho_P}{\mu} T, \tag{2}$$

where μ is the exciton mass. In these equations the nonlinear term $(\mathbf{v}\nabla)\mathbf{v}$ was dropped and the Clapeyron equation is valid. The exciton temprature T in the same approximation is assumed to be equal to the lattice temperature.

These equations have the following boundary conditions: a) the pressure of the supersaturated exciton gas p_∞ produced by the laser is constant at a distance from the drop, b) the pressure at the surface of the drop is equal to that of the saturated vapor, if the curvature of the surface is taken into account[2] (R_1 and R_2 are the main radii of the curvature and σ is the surface tension),

$$p_P|_S = p_0(T), \qquad p_L|_S = p_0(T) + \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$

c) the normal mass flow at the surface of the drop is continuous and the motion of the drop is taken into account.

Equations (1) and (2) with the specified boundary conditions have a spherically symmetric, steady-state solution:

$$p_P^o = p_0(T)e^{-U/T} + \frac{\rho_L R_0^3}{3\tau_P \tau_0} e^{-U/T} \int_{R_0}^{r} \frac{1}{r^2} e^{U/T} dr,$$

$$p_L^o = -n_L U + \frac{\rho_L r^2}{6\tau_0 \tau_L} + \text{const}. \tag{3}$$

The radius of the drop is determined by

$$p_\infty - p_0(T) = \frac{\rho_L R_0^3}{\tau_L \tau_0} \int_{R_0}^{\infty} \frac{1}{r^2} e^{U/T} dr .$$

In analyzing the stability of this solution, we assumed that

$$p = p^o(r) + \sum_{l,m} \exp(\lambda_{lm} t) Y_{lm}(\theta, \phi) p_{lm}(r).$$

The shape of the drop is given by

$$R = R_0 + \sum_{lm} \exp(\lambda_{lm} t) Y_{lm}(\theta, \phi) \zeta_{lm}.$$

The variables are divided in the spherical coordinates (Y_{lm} are spherical functions) and the linearized equations have the form

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2\left(\frac{\partial p_{lm}}{\partial r} + \frac{p_{lm}}{T}\frac{\partial U}{\partial r}\right) - \frac{l(l+1)}{r^2} p_{lm} = 0 \tag{4}$$

in the pair and an analogous equation with $U = 0$ in the EHL. The boundary conditions can be written in the form

$$\left(p_{lm} + \frac{\partial p^o}{\partial r}\zeta_{lm}\right)_{R_0+0} = 0, \left(p_{lm} + \frac{\partial p^o}{\partial r}\zeta_{lm}\right)_{R_0-0} = \frac{\sigma(l-1)(l+2)}{R_0^2}\zeta_{lm}.$$

$$p_{lm}|_{r\to\infty} = 0$$

$$\left[-\tau_L \frac{\partial p_{lm}}{\partial r} + \frac{\partial(\rho_L v_L)^o}{\partial r}\zeta_{lm}\right]_{R_0-0} - \rho_L \lambda \zeta_{lm} = \left[-\tau_P\left(\frac{\partial p_{lm}}{\partial r} + \frac{p_{lm}}{T}\frac{\partial U}{\partial r}\right)\right.$$

$$\left. + \frac{\partial(\rho_P v_P)^o}{\partial r}\zeta_{lm}\right]_{R_0+0}. \tag{5}$$

We used a well-known expression for $(1/R_1)+(1/R_2)$ at the nearly spherical surface (see, for example, Ref. 3). The quantity m is not included in Eqs. (4) and (5), so that the eigenvalues λ_{lm} are independent of m and are $2l+1$-fold degenerate. We ignored the terms with λ, which take into account the compressibility, under the assumption that $\lambda \ll \tau_{\Pi} T/\mu R^2$. Since the linearized equations can be easily solved at $U=0$, we write the corresponding expression for λ_l

$$\lambda_l = (l-1)\left[\frac{2}{3\tau_0} - \frac{\sigma\tau_L}{\rho_L R_0^3}\, l(l+2)\right].$$

Thus, the stability criterion has the form:

$$R_0 \lessgtr R_c = \left(\frac{12\tau_L\tau_q\sigma}{\rho_L}\right)^{1/3}.$$

As expected, the instability initially sets in at $l=2$. According to the existing data for germanium,[4] $\sigma \approx 10^{-4}$, $\tau_0 \approx 10^{-5}$, $\tau_L \approx 10^{-9}$, and $n_L \approx 10^{17}$ (CGS); hence, $R_c \approx 10^{-3}$ cm. Note that these data are very approximate. Because of the degeneracy of m, we cannot determine from general considerations when the λ's are small whether the drops will be nonspherical and stationary or whether they will break up immediately (hard excitation).

The case of $U \neq 0$ is more complicated. Generally, λ_l can be determined only by numerical integration. We examined a model in which we assumed that $rU'/T \gg 1$, $r<r_1$ and $rU'/T \ll 1$, $r>r_1>R_0$. This model makes it possible to determine the analytic expression for solution of the equations for the exciton pair.[5] The expression for perturbation of the vapor pressure has the form

$$p_l = \frac{C}{r^{l+1}}, \quad r>r_1; \quad p_l = Ae^{-U/T} + \frac{BT}{r^2U'}, \quad r<r_1.$$

The quantities A and B are slowly varying functions.

Omitting simple calculations, we write the expression for the increment:

$$\lambda_l = \frac{l-3}{3\tau_0} - l\frac{\partial U}{\partial R_0}\frac{n_L\tau_L}{\rho_L R_0}$$

(the small terms in the parameters $T/U'R$ and ρ_π/ρ_L were dropped). The stability condition has the form:

$$\frac{\partial U}{\partial R_0} > \frac{\rho_L}{n_L}\frac{R_0}{\tau_0\tau_L}.$$

Note that in case of instability the perturbations with large $l>3$ now begin to increase; moreover, the higher the number the faster they increase (if the surface tension is ignored), which apparently immediately breaks up the drop.

The physical nature of the specified instability is attributed to the formation in places of larger surface curvature of the drop of large concentration gradients of excitons, which produce a large diffusion flux. This large diffusion flux draws the surface of the drop toward itself and further increases the original curvature.

In conclusion, we note that, although there is no absolute certainty that the real size of the drops is connected with the proposed instability mechanism, its existence must play a role in different, more refined theories of the structure and formation of EHD.

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