Stability of trough oscillations in ambipolar traps

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It is shown that the trough oscillations of plasma in ambipolar traps can be stable even if the magnetic field is axially symmetric, i.e., devoid "min-B" on the axis of the system. A low-pressure plasma with potential oscillations is examined.

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1. In ambipolar traps[1] the large loss of particles through the ends, the main deficiency of open magnetic traps, has been overcome. The purpose of this paper is to show that such systems can also stabilize the trough instability. The configurations of the magnetic field with "min-B" are usually used in this case. These configurations, however, are technically difficult to achieve and the unavoidable disturbance of the axial symmetry of the magnetic field opens a new channel of loss.[2] We show that the ambipolar traps can be stabilized against trough oscillations without using the configurations with "min-B."

2. The ambipolar traps are systems comprised of three open traps connected in series; the number of particles contained in the outer traps $N^{(1)}$ and the characteristic variation of the magnetic field $L^{(1)}$ are much smaller than the corresponding values for the average trap $N^{(0)}$ and $L^{(0)}$ and the average energy of the ions $\epsilon_i^{(1)}$ is much higher than $T_i^{(0)}$ and T_e. In this paper, the values for the average trap are denoted by "0," those for the outer traps are denoted by "1," and the values characterizing the electronic component have no index, since the electrons are collectivized in such systems.

The increment of the trough instability, which is produced in the ordinary axially symmetric traps, greatly exceeds the angular velocity of the azimuthal ion drift due to the nonuniformity of the magnetic field, although such a drift is the "motive force" of the instability. The large central part of the ambipolar traps increases the inertia of the system, which decreases the increment. If the increment

$$\gamma \approx \omega_i^{(0)} \frac{\rho_i^{(1)}}{L^{(1)}} \left(\frac{N^{(1)}}{N^{(0)}} \right)^{1/2},$$

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which is determined according to the traditional scheme (see, for example, Refs. 3 and 4), is smaller than the drift velocity $\omega_i^{*(1)} \approx \omega_i^{(1)} (\rho_i^{(1)}/L^{(1)})^2$, then this scheme, which is based on the expansion over the parameter $\omega_i^{*(1)}/\omega$ is inadequate. Here $\rho_i^{(1)}$ is the Larmor radius of the ions and $\omega_i^{(1)}$ is the cyclotron frequency.

The differential equation for the trough oscillations, which is valid at $\omega \ll \omega_i^{(1)}$, has the form

$$\frac{1}{r}\frac{d}{dr} S \frac{d\psi}{dr} + \frac{1-m^2}{r^3} S\psi + (m^2\gamma^{(o)2} + \omega^2) r \frac{dn^{(o)}}{dr}\psi$$
$$- m\delta\omega\omega_i^{(o)} \left\langle \frac{m\omega_i^{*(1)}}{\omega - m\omega_i^{*(1)}} \right\rangle r \frac{dn^{(o)}}{dr}\psi = 0 \qquad (1)$$

It was obtained by averaging the Poisson equation over the tube of the magnetic lines of force (trough), which passes through the whole system. To determine the perturbations of the charge density, we used the methods of Timofeev.[4] In Eq. (1) $\psi = (1/r)\phi$, ϕ is the perturbation of the potential, m is the azimuthal wave number, r is the radius of the field tube in the central trap,

$$S = \omega^2 r^3 \left(1 - \frac{m\omega_L^{(0)}}{\omega}\right) n^{(0)}, \quad \omega_L^{(0)} = \frac{T_i^{(0)}}{m_i \omega_i^{(0)}} \frac{1}{rn^{(0)}} \frac{dn^{(0)}}{dr}$$

is the angular velocity of the Larmor drift of the ions,

$$\gamma^{(0)2} = \frac{T_i^{(0)} + T_e}{m_i L^{(0)2}} \frac{H^{(1)}}{H^{(0)}}, \quad \delta = N^{(1)}/N^{(0)},$$

and the angle brackets $\langle ... \rangle$ denote averaging over the velocity distribution function of the ions in the outer traps.

In $n^{(0)}(r) = n_0^{(0)} e^{-r^2/a^{(0)2}}$, then the solution of Eq. (1) can be expressed in terms of the degenerate hypergeometric functions[3,4] and the dispersion equation for the self-induced oscillation frequency has the form

$$\omega^2 + m^2\gamma^{(o)2} - m\delta\omega\omega_i^{(o)} \left\langle \frac{m\omega_i^{*(1)}}{\omega - m\omega_i^{*(1)}} \right\rangle + A_{m,n}\omega(\omega - m\omega_L^{(0)}) = 0. \qquad (2)$$

where $A_{m,n} = n + (1/2)(m-1)$ and n is the radial wave number.

3. Let us first examine the stability of the so-called first mode, ($n = 0$, $m = 1$, $A_{m,n} = 0$). If the velocity distribution of the ions is a δ function, then Eq. (2) is a third-degree algebraic equation. We can easily show that if the following conditions are fulfilled $\omega_i^{*(1)} \gg \delta\omega_i^{(0)} > 2\gamma^{(0)}$ or in a different form

$$\frac{H^{(1)}}{H^{(0)}} \left(\frac{\rho_i^{(1)}}{L^{(1)}}\right)^2 \gg \frac{N^{(1)}}{N^{(0)}} > 2\frac{\rho_i^{(0)}}{L^{(0)}} \left(\frac{H^{(1)}}{H^{(0)}}\right)^{1/2},$$

then the first mode will be stable, irrespective of the sign of $\omega_i^{*(1)}$. (The ordinary traps are stable only if the magnetic field increases along the radius $\omega_i^{*(1)} > 0$.) Thus, at first glance we draw a paradoxical conclusion: three traps, each of which is individually unstable, become stabilized when combined. The stablization mechanism resembles the stabilization mechanism due to the effects of the finite Larmor radius.[3] As is well known, the effects of the finite Larmor radius slow down the ion drift in the perturbations of the potential. As a result, the electrons are displaced a large distance along the radius and the perturbation of the electron density exceeds that of the ions. The electric field of the excess charge impedes the plasma discharge into the region of smaller magnetic field, which stabilizes the oscillations. In this case the excess electronic charge is attributed to the fact that all the electrons participate in the low-frequency oscillations with $\omega_{1,2} \approx \frac{1}{2}\{ -\delta\omega_i^{(0)} \pm [(\delta\omega_i^{(0)})^2 - 4\gamma^{(0)2}]\}^{1/2}$ but only those ions which are in the central trap.

The high-frequency oscillations with $\omega_3 \approx \omega_i^{*(1)}$, however, are stable, since they affect neither the electronic component nor the ions in the central trap. Even a small spread in the velocity distribution of ions eliminates this oscillation branch.[5] At the same time, this spread affects weakly the low-frequency oscillations. In fact, the ambipolar electric field ejects from the outer traps the ions whose energy is smaller than the average energy by several (3–5) factors. Therefore, the drift velocity of all the ions in the outer traps must greatly exceed ω.

If the velocity distribution of ions is a δ function, then all the other modes will be stabilized along with the first mode. However, one of the oscillation branches may turn out to be unstable, if there is a spread in the ion distribution. The buildup of oscillations is caused by the resonance interaction with the ion drift in the outer traps. This can be easily verified if we take into account the imaginary part of the third term in Eq. (2) and use the relation

$$\operatorname{Im} \frac{1}{\omega - m\omega_i^{*(1)}} = -\pi\delta(\omega - m\omega_i^{*(1)})$$

in averaging over the velocities. The instability increment can increase to approximately $\delta\omega_i^{(0)}$. To stabilize the oscillations one of the inequalities $B = (\omega_L^{(0)}/\omega_i^{*(1)}) \ll 1$ or $B \gg 1$ must be satisfied. Using the relation $H^{(0)}a^{(0)2} = H^{(1)}a^{(1)2}$, we can represent B in the form $(T_i^{(0)}/\epsilon_i^{(1)})[(L^{(1)}/a^{(1)})]^2$. In the first case the ions, which can resonate with the oscillations, are not held by the trap (see above) and in the second case their number is exponentially small.

In conclusion, we note that the stability conditions established by us do not impose impracticable conditions on the parameters of the hypothetic thermonuclear reactors.[1,6]

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