Boundary conditions and generation of periodic noise by a space-charge wave

L. P. Gor'kov
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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The excitation of oscillations by a moving space-charge wave is attributed to a nonlinear periodic regime which arises near a contact with a normal conductor. This regime corresponds to either a homogeneous phase slippage (thin samples) or a space-charge wave which is moving across the track of edge dislocations in the lattice. Some simple microscopic equations are proposed.

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The excitation of nearly periodic oscillations by a space-charge wave was observed originally in $NbSe_3$ (Refs. 1 and 2) and later in TaS_3 (Ref. 3) and $K_{0.3}MoO_3$ (Ref. 4). Most of the corresponding models[5–14] interpret the effect on the basis of a nonlinear equation for the phase of the space-charge wave in the presence of a pinning potential, in an effort to relate the excitation effect to the presence of a threshold electric field (E_t) required for motion of the wave. In the present letter we argue that when the pinning of the space-charge wave is a volume effect (defects[15]) the excitation of periodic oscillations results from the boundary conditions near a contact with a normal conductor, which give rise to phase-slippage centers. This mechanism, from the theory of superconductivity,[16,17] requires incorporating the change in the modulus of the order parameter.

Space-charge waves are described by a lattice strain $u(x) = u_0 \cos(2k_F x + \phi)$, where an arbitrary phase in the incommensurate case reflects the possibility that the wave can move in the absence of pinning. We will ignore the latter ($E > E_t$). As the order parameter

$$\Delta(x, t) = |\Delta(x, t)| \exp i\phi(x, t) \tag{1}$$

in the slowly moving fields we adopt the size of the "gap" in the electron spectrum, which is proportional to the local strain.

A volume strain (1) can occur only below the transition temperature T_p, while a large strain (large on the atomic scale) is unavoidable near an interface between two media. This strain is relatively insensitive to the low-temperature structural transition. If it were not, a structure appearing spontaneously in the volume at $T < T_p$ would have to change over a distance $\xi_0 \sim \hbar v/T_p$ to suit the existing strain at the boundary. This condition unavoidably fixes the phase of the space-charge wave at the boundary.

Assuming that the change $|\Delta|$ is small in a weak field, we find

$$\Gamma\phi = aE + \xi_0^2 \nabla^2 \phi \,. \tag{2}$$

Ordinarily (the relaxation time τ is set by impurities) we would have[18,19]

$$a \sim e\xi_0/T_p, \quad \Gamma^{-1} \sim T_p^2 \tau . \tag{3}$$

A steady-state ($\dot{\phi} = 0$) solution of (2) can occur in the volume only in the presence of pinning forces; in the opposite case, vector (1) rotates at a frequency

$$\Omega = a\Gamma^{-1} E \sim eEl , \tag{4}$$

corresponding to the motion of a space-charge wave at a velocity $2k_F u = \Omega$.

Figure 1 is a mechanical model described by (2). A spring fixed at $x = 0$ is rotated at a distant point at the angular velocity Ω given by (4). A steady-state process can be established if, in some cross section $x = x_0$, the coupling between two regions is periodically broken because of the accumulated stress, and a phase slippage $[\phi] = 2\pi$ occurs. In parabolic equation (2) the term with the field is a spatially homogeneous source, while the slippage process provides a phase "sink": $2\pi\Gamma^{-1}\delta(x - x_0)\cdot\delta(t - t_n)$, where $t_n = 2\pi\Omega^{-1}n$.

The arguments can be supported by a convenient microscopic model. We assume that the electron spectrum $\epsilon(\mathbf{p})$ is three-dimensional and corresponds to two regions with the property of "superposition,"[20] $\epsilon(\mathbf{p} + \mathbf{q}) = -\epsilon(\mathbf{p})$ (for example, tunneling between chains corresponds to the strong-coupling approximation[21]). If the longitudinal component $q_{\|} = 2k_F$ is incommensurate, the metal-dielectric transition gives rise to superstructure (1). Particularly simple equations arise if the transition temperature is lowered ($T_p \ll T_{p0}$) by defects, i.e., in the range of concentrations below the critical value $1/\tau_c = 4\gamma/3\pi T_{p0}$:

$$\begin{aligned} &\Delta + (\Delta^2 + Q^2 - 1)\Delta - \nabla^2\Delta = 0 , \\ &(E + \phi)\Delta^2 - \operatorname{div}(Q\Delta^2) = 0 \end{aligned} \tag{5}$$

(where $\Delta \equiv |\Delta|, Q = \widehat{\nabla\phi}$). The order parameter in (5) is expressed in units of $\Delta_\infty = \pi T_p \delta^{1/2}$ ($\delta = 1 - T^2/T_p^2$); the field eE is expressed in units of $\pi^2 T_p^2 \delta/12\bar{v}$; and the times and coordinates are expressed in units of $9/2\pi^2 T_p^2 \tau_c \delta$ and $\sqrt{3\bar{v}^2/2}/\pi T_p$, respectively. The current is carried primarily by normal carriers,

$$j = \frac{\sigma_n \pi^2 T_p^2}{12 e \bar{v}} \left\{ E - \lambda E \Delta^2 - \left(\frac{8}{9}\right) \lambda \epsilon \Delta^2 \dot{\phi} \right\} . \tag{6}$$

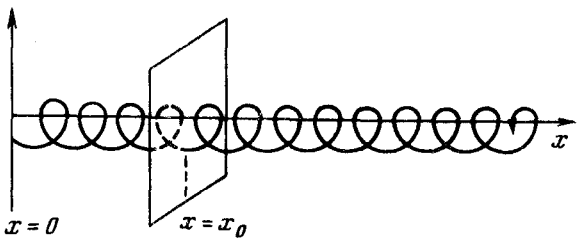

FIG. 1.

The second term in (6) is a small correction to the normal conductivity caused by the transition, while the third term reflects the moving space-charge wave ($\lambda = (8\pi^2/3)T_p^2\tau_c^2\delta$, $\epsilon = \bar{v}^2/\overline{v^2}$).

The point x_0 in Fig. 1 is determined by integrating nonlinear equations (5) and using the conditions $\Delta = \Delta_0$ and $\phi = 0$ at $x = 0$ and $\Delta = 1$ and $\dot{\phi} = -E$ in the limit $x \to \infty$. The choice of x_0 is not obvious. At $E \ll 1$ we could have a regime with $x_0 \gg 1$. At $x < x_0$, all quantities would then be periodic in the time, and Δ would vary slowly along the coordinate. For the average quantities we find from (5)

$$\bar{\Lambda}^{-2} = 1 - \bar{Q}^{\,2}\,; \quad 3\bar{Q} - 2\,\mathrm{Arcth}\,\bar{Q} = Ex\,.$$

Corresponding to the stable branch of the solutions of the second equation is the value $\bar{Q} < 1/3$:

$$Ex_0 < 1 - \ln 2\,. \tag{7}$$

This example shows that an electric field creates near the boundary a phase gradient, i.e., a local change in the period of the space-charge wave. This change increases along the coordinate (and over time) until the superstructure loses its stability. At distances $|x - x_0| \sim 1$, rapid changes occur in all quantities, and Δ crosses zero.

At $E \gg 1$ the solution yields $x_0 \cong (2E)^{-1/2} \ln E$. The amplitude of the variable stress ($j = \mathrm{const}$) is

$$e\,V_1 = \left(1 - \frac{8}{9}\epsilon\right)\frac{2\pi^3}{3\sqrt{3}}\,\epsilon^{1/2}\,\delta(T_p\tau_c)^2\,T_p\,\sin(24\,eElt)\,.$$

In a sample which is unbounded in the transverse direction a phase sink can occur in the x_0 plane because of the motion of a periodic structure of the phase-slippage centers if the period L and velocity $v_\perp$ of the structure are related by $2\pi v_\perp = \Omega L$. An estimate from (5) would give us $v_\perp \sim v(T_p\tau_c)\delta$. Physically, this structure would represent edge dislocations in the lattice of the space-charge wave. Which solution to choose (that corresponding to the homogeneous phase-slippage center or that corresponding to a traveling track) is not clear. Fung and Steeds[23] have shown that an $NbSe_3$ single crystal consists of normal domains ($\sim 10^{-6}$ cm wide) oriented parallel to the principal axis; this situation would seem to further hinder the motion of dislocations.

According to (4), the frequency of the oscillations which are excited is a measure of the mean free path in actual substances. In turn, working from (2) and (3) we find the following criterion according to result (7) (under the condition $\xi_0 Q \sim 1$):

$$eEx_0 \sim T_p\,.$$

We then find $x_0 \sim 10^{-2}$ cm (with $E \sim 100$ mV/cm and $T_p \sim 50$ K). In other words, x_0 is approximately equal to the length $l \sim 100\,\mu$m found in Ref. 22 from an estimate of the effect of the sample geometry.

It should be noted that the concept of phase-slippage centers was recently proposed independently in an experimental study by Ong and Verma[24] in an effort to explain data on the local nature of the excitation mechanism in $NbSe_3$.

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