Parametric excitation of surface waves in a semi-infinite plasma

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A new parametric-instability branch has been found for a semi-infinite plasma in a weak rf field. This instability occurs over a broader range of frequencies of the external field than the instability branch identified previously.

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The threshold external field for parametric instabilities involving surface waves in a bounded plasma is much higher[2] than that for the parametric excitation of plasma waves in an infinite plasma.[1] The reason for the difference lies in the rapid Landau damping of an rf surface wave,[3] with a damping rate higher than the frequency of ion acoustic surface waves. The parametric interaction of an rf field with a bounded plasma is therefore caused by the excitation of quasistatic rf surface waves and of some low-frequency waves which are not natural waves of the plasma.[4] A detailed study of the parametric instability of a bounded plasma in an alternating electric field is important for correctly interpreting experiments,[5] for determining the conditions under which the effect can be exploited, and for determining the nature of the parametric excitation of surface waves in a semi-infinite nonisothermal plasma.

The particular characteristics of the parametric instability of a bounded plasma stem from the circumstance that this instability can proceed by two paths. The first is closely associated with the existence of a plasma boundary.[4] The second arises from the interaction of the rf and low-frequency waves in the nonuniform pump field. To prove this assertion is the purpose of the present paper.

We put the interface between the plasma and a dielectric in the xy plane. The plasma fills the half-space $z \geqslant 0$, while the dielectric fills the half-space $z < 0$. We work from the equations of two-fluid quasihydrodynamics and Maxwell's equations. The electron temperature T_e is considerably higher than the temperature of the singly charged ions, T_i; without any loss of generality, we can set the latter temperature equal to zero. An alternating electric field $\mathbf{E}_0 = (E_{0x}, 0, 0)$ is applied to the plasma boundary. This field varies over the time t at a frequency ω_0 which is approximately equal to the limiting frequency of quasistatic surface waves, $\omega_{pe}(1+\epsilon_d)^{-1/2}$, where ϵ_d is the static dielectric function of the dielectric, and ω_{pe} is the electron plasma frequency. Linearizing the original system of equations with respect to small amplitudes of the waves that are excited, we find the following system of equations and boundary conditions for the field amplitudes of the fluctuational waves:

$$(\Delta_\perp - \kappa^2)\,\mathbf{H}^{(\pm *)} = g_{(\pm)}\,\mathbf{U}^{(\pm *)}, \quad (\Delta_\perp - \kappa_d^2)\,\mathbf{H}_d^{(\pm *)} = 0, \quad \operatorname{div}\mathbf{H}^{(\pm *)} = 0 \qquad (1)$$

$$U_z^{(\pm *)} = k_y \frac{\omega_{pe}^2}{\omega_0 c} \widetilde{\delta n}^{(+)} E_0^{(*)},$$

$$U_y^{(\pm *)} = i \frac{\omega_{pe}^2}{\omega_0 c} E_0^{(*)} \nabla_\perp \widetilde{\delta n}^{(+)} + i \frac{\omega_0 \omega_{pe}^2}{c^3} \frac{\epsilon_0}{\kappa_0} E_0^{(*)} \widetilde{\delta n}^{(+)}, \; U_x^{(\pm *)} = 0 ,$$

$$\{\frac{1}{\epsilon} (\nabla_\perp H_y^{(\pm *)} + ik_y H_z^{(\pm *)})\} = i g_{(\pm)} \frac{\omega_{pe}^2}{\omega_0^2 \epsilon_0} \widetilde{\delta n}^{(+)}(0) E_0^*,$$

$$\{\frac{1}{\epsilon} (\nabla_\perp H_x^{(\pm *)} + ik_x H_z^{(\pm *)})\} = \{H_{y,x}^{(\pm *)}\} = 0 , \qquad (2)$$

$$(\Delta_\perp - \kappa_s^2) \widetilde{\delta n}^{(+)} = - i \frac{e^2 c^2 \kappa}{2 m_e T_e \omega_0^3 \epsilon_0} (\Delta_\perp - k^2)(H_y^{(+)} E_0^* - H_y^{(-*)} E_0), \qquad (3)$$

$$(\Delta_\perp - k^2)\phi^{(+)} = - \frac{m_i}{e} \Omega^2 \widetilde{\delta n}^{(+)}, \quad (\Delta_\perp - k^2)\phi_d^{(+)} = 0 \qquad (4)$$

$$\{\phi^{(+)}\} = 0, \quad \nabla_\perp \phi^{(+)}|_{z=+0} = 0 , \qquad (5)$$

where $\mathbf{H}^{(\pm *)}$ and $\mathbf{H}_d^{\pm *}$ are the magnetic field amplitudes of the rf waves in the plasma and in the dielectric. The superscripts + and − specify the waves which are propagating in the positive and negative directions in the $z = \text{const}$ plane, while the asterisk denotes the complex conjugate. Fourier transforms have been taken in the variables x, y, and t; Ω is the frequency of the low-frequency wave; and the rest of the notation is explained by

$$\kappa^2 = k^2 - \omega_0^2 \epsilon_0 / c^2, \quad \kappa_d^2 = k^2 - \omega_0^2 \epsilon_d / c^2,$$

$$\kappa_0^2 = - \omega_0^2 \epsilon_0 / c^2 \quad \kappa_s^2 = k^2 - \Omega^2 / c_s^2 , \; g_{(\pm)} = \pm 1 ,$$

$$c_s^2 = T_e / m_i , \qquad \epsilon_0 = 1 - (\omega_{pe}^2 / \omega_0^2). \qquad k^2 = k_x^2 + k_y^2 , \; \nabla_\perp = \partial/\partial z, \qquad \Delta_\perp = (\nabla_\perp)^2.$$

The braces denote the discontinuities in the enclosed quantities at the $z = 0$ interface; $\delta n^{(+)} = n_0 \delta n^{(+)}$ is the variation of the plasma density in the low-frequency wave; and $\phi^{(+)}$ and $\phi_d^{(+)}$ are the potentials in the plasma and in the dielectric. Solving Eqs. (1), and using boundary conditions (2), we find expressions for the fields of the rf surface waves in terms of the pump field and the density variation $\delta n^{(+)}$. Substituting these expressions into Eq. (3), we convert it to

$$\widetilde{\delta n}^{(+)}(z) + \lambda a(z) \int_0^\infty \widetilde{\delta n}^{(+)}(s) a(s) ds = C_1 \exp(-\kappa_s z) , \qquad (6)$$

where

$$\lambda = (1+\epsilon_d)^{1/2} \frac{|E_0|^2}{16\pi n_0 T_e} \frac{(\kappa_0+\kappa)^2 - k^2}{(\kappa_0+\kappa)^2 - \kappa_s^2} k_x^2 \left[\frac{\omega_{pe}}{\delta + \delta_k + \Omega - i\tilde{\gamma}} + \frac{\omega_{pe}}{\delta + \delta_k - \Omega + i\tilde{\gamma}} \right] \frac{1}{k}$$

$$a(z) = \exp[-(\kappa_0+\kappa)z]$$

$$\delta = \omega_0 - \omega_{pe}(1+\epsilon_d)^{-1/2}, \quad \delta_k = \frac{\omega_{pe}^3 \epsilon_d^2}{2k^2c^2(1+\epsilon_d)^{5/2}}, \quad \tilde{\gamma} = \frac{k}{\sqrt{\pi}} \sqrt{T_e/m_e}$$

and C_1 is an arbitrary constant. System of equations (1) and (3), which is a system of coupled second-order differential equations, has thus been reduced to a single Fredholm integral equation with a self-adjoint kernel $a(z)a(s)$. We assume that λ is not a characteristic number of the homogeneous integral equation corresponding to Eq. (6); then Eq. (6) has a unique solution for an arbitrary value of the constant C_1:

$$\widetilde{\delta n}^{(+)} = C_1 \left[\exp(-\kappa_s z) - \frac{\lambda a(z)}{\Delta(\kappa_0+\kappa+\kappa_s)} \right], \quad \Delta = 1 + \frac{\lambda}{2(\kappa_0+\kappa)} \neq 0. \tag{7}$$

We find the potential $\phi^{(+)}$ from Eq. (4), using (7):

$$\phi^{(+)} = -\frac{m_i}{e} \Omega^2 C_1 \left[\frac{\exp(-\kappa_s z)}{\kappa_s^2 - k^2} - \frac{\lambda a(z)}{\Delta(\kappa_0+\kappa+\kappa_s)[(\kappa_0+\kappa)^2 - k^2]} \right]$$

$$+ C_2 \exp(-kz), \quad \phi_d^{(+)} = C_3 \exp(kz).$$

We set[1] $C_2 = 0$ and substitute $\phi^{(+)}$ and $\phi_d^{(+)}$ into boundary conditions (5). As a result, we find the dispersion relation of Ref. 4:

$$\frac{\kappa_s(\kappa_0+\kappa+\kappa_s)}{\kappa_s^2 - k^2} = \lambda[(\kappa_0+\kappa)^2 - k^2]^{-1}(\kappa_0+\kappa)\Delta^{-1}. \tag{8}$$

We now assume that λ is equal to a characteristic number of the homogeneous version of Eq. (6).

$$\lambda = \lambda_* = -2(\kappa_0+\kappa). \tag{9}$$

The eigenfunctions of the kernel $a(z)a(s)$ corresponding to the characteristic number λ_* are

$$\widetilde{\delta n}_*^{(+)} = C_4 \lambda_* a(z). \tag{10}$$

Since eigenfunctions (10) are not orthogonal to the right side of Eq. (6), they are solutions of this equation only if $C_1 = 0$. We assume $C_1 = 0$. Substituting (10) into (4), we find the potential

$$\phi_*^{(+)} = -\frac{m_i}{e}\Omega^2 C_4 \frac{\lambda_* a(z)}{(\kappa_0 + \kappa)^2 - k^2} + C_{2*}\exp(-kz),\quad \phi_{d*}^{(+)} = C_{3*}\exp(kz).$$

Boundary conditions (5) determine the relationship among the constants C_4, C_{2*}, and C_{3*}. The solutions for the dimensionless variation of the plasma density, $\delta n_*^{(+)}$, and for the potential $\phi_*^{(+)}$ are not eigenfunctions for either the plasma or the plasma-dielectric interface. These solutions therefore hold only if dispersion relation (9) has solutions with $\mathrm{Im}\Omega < 0$ (an instability). In the opposite case ($\mathrm{Im}\Omega \geqslant 0$), we should set $C_4 = 0$. Analysis of dispersion relations (8) and (9) shows that the threshold external fields for the instabilities with the growth rates γ_1 and γ_2 found from Eqs. (8) and (9), respectively, are the same:

$$\frac{|E_0|^2}{32\pi n_0 T_e} = \frac{\tilde{\gamma}}{\omega_{pe}}(1+\epsilon_d)^{-1/2}\frac{k^2}{k_x^2}.$$

The maximum growth rates $\gamma_{1\,\max}$ and $\gamma_{2\,\max}$ at field levels well above the threshold are

$$\gamma_{1\,max} = (1+\epsilon_d)^{1/4}|k_{1x}|C_s\left(\frac{\omega_{pe}}{\tilde{\gamma}}\right)^{1/2}\left(\frac{|E_0|^2}{16\pi n_0 T_e}\right)^{1/2}\left(1-\frac{\kappa_0}{k_1}\right)^{1/2},$$

$$\delta + \delta_{k1} = -\tilde{\gamma};$$

$$\gamma_{2max} = (1+\epsilon_d)^{1/4}|k_{2x}|C_s\left(\frac{\omega_{pe}}{\tilde{\gamma}}\right)^{1/2}\left(\frac{|E_0|^2}{16\pi n_0 T_e}\right)^{1/2}\left(\frac{\kappa_0}{k_2}\right)^{1/2},$$

$$\delta + \delta_{k2} = \tilde{\gamma}.$$

The equations $\delta + \delta_{k_1,2} = \mp\tilde{\gamma}$ determine the wave numbers of the rf surface waves. Taking $T_e = 1$ eV, $\epsilon_d = 10$, $(kc/\omega_{pe}) = 2.5$, we find $\gamma_1 = 1.2\,\gamma_2$; i.e., the quantitative difference between γ_2 and γ_1 is small. The primary distinction between the two instabilities is that the surface waves with the growth rate γ_2 can be excited either at $\omega_0 \lesssim \omega_{gr}$ or at $\omega_0 \gtrsim \omega_{gr}$, while the instability with γ_1 can occur only if $\omega_0 \lesssim \omega_{gr} \equiv \omega_{pe} \times (1+\epsilon_d)^{-1/2}$.

[1)]The condition $C_2 = 0$ is required for the limiting transition to ion acoustic dispersion at a zero amplitude of the pump field.

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