

Interaction between localized magnetic moments in disordered conductors

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It is shown that because of diffusion of conduction electrons, the interaction between them causes the indirect exchange between localized moments to decrease in a power-law fashion at distances exceeding the mean free path. In the case of repulsion between electrons, it has the ferromagnetic sign, so that when the density of localized spins increases, a transition from the ferromagnetic state to the spin-glass state occurs.

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In conductors, the basic mechanism of interaction between localized spin moments is RKKI indirect exchange. The dependence of this interaction on the distance between the localized magnetic moments is determined by the behavior of the electron-spin correlation function:

$$K(\mathbf{R}) = \int_0^{T^{-1}} d\tau \langle \sigma_z(\mathbf{r}_1\tau) \sigma_z(\mathbf{r}_2) \rangle, \quad (1)$$
$$\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2.$$

The interaction Hamiltonian for two spins is

$$H = -J^2 K(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{S}_1 \cdot \mathbf{S}_2 \quad (2)$$

where J is the interaction constant of a localized spin interacting with the conduction electron.

For noninteracting electrons, $K(\mathbf{R})$ at distances R less than the mean free path of electrons l oscillates with the period $1/p_F$ ($\hbar \equiv 1$), where p_F is the Fermi momentum of the electron, and decreases as R^{-3} . At distances greater than the mean free path, $K(R)$ begins to decrease exponentially as $\exp\{-R/l\}$.

It is shown in Ref. 1 that the spin susceptibility of conduction electrons exhibits spatial dispersion. We shall show here that the diffusion nature of the motion of interacting electrons leads to the fact that the correlation of the spins of conduction electrons remains at distances greater than the mean free path. In addition, the electron-spin correlation function begins to decrease exponentially only when $R > L_0 \equiv \min\{L_T, L_s\}$, where $L_T = \sqrt{D/T}$, $L_s = \sqrt{Dt_s}$, T is the temperature, D is the diffusion coefficient of electrons, and t_s is the electron-spin relaxation time.²

We should point out the following situation:

1) For $lk < R < L_0$, $K(\mathbf{R})$ decreases in a power-law fashion and in the three-dimensional case

$$K(\mathbf{R}) = \frac{\lambda}{8\pi^3 DR^4}. \quad (3)$$

Here λ is the dimensionless electron-electron interaction constant.

2) For $R > l$, $K(\mathbf{R})$ is monotonic.

3) The exchange interaction, given by expression (2), has for $|\mathbf{r}_1 - \mathbf{r}_2| > l$ the ferromagnetic sign in the case of repulsion between electrons, when $\lambda > 0$.

Thus we can expect that in spin ordering in disordered conductors, in which the localized spin density N_s is such that the average distance between spins is $R_s = [(3/4\pi)N_s]^{-1/3} > l$, is ferromagnetic in nature and that a spin glass will form as the density is increased, with $R_s < l$.

The Curie temperature T_C is determined by the magnitude of the exchange interaction. In the three-dimensional case, with $R_s > l$ it follows from (2) and (3) that

$$T_K \cong \frac{S(S+1)}{3} J^2 K(R_s) = \frac{S(S+1)}{24\pi^3} \frac{\lambda J^2}{DR_s^4}. \quad (4)$$

In order of magnitude, $T_K \sim \lambda (Jv_0)^2 \tau^{-1} (N_s/N)^{4/3}$, where $v_0 = mp_F/2\pi^2$ is the density of states at the Fermi level, τ is the momentum relaxation time of electrons for elastic collisions with normal impurities, defects, etc., and N is the electron density.

We estimated the transition temperature as the value of the exchange constant at the average distance between localized spins, which is valid to within terms of order one.

Expressions (3) and (4) are valid for three-dimensional specimens, i.e., for specimens whose geometrical dimensions are greater than L_0 . If one of the dimensions of the specimen is less than L_0 , then the effective dimensionality of the specimen decreases, and the effect under study becomes more important. Thus, in a film whose thickness $a < L_0$, electron diffusion has a two-dimensional nature. In this case, at distances $R > a$,

$$K(R) = \frac{\lambda}{\pi^3 D a^2 R^2}. \quad (5)$$

Because of the long-range nature of the exchange interaction, spins located not only at the average distance but also at greater distances contribute to T_C . For this reason,

$$T_K \cong \frac{S(S+1)}{3} J^2 N_s a \int_{R_s < R < L_s} d^2\mathbf{R} K(\mathbf{R}) = \frac{2S(S+1)}{3\pi^2} \lambda \frac{J^2 N_s}{Da} \ln \frac{L_s}{R_s}. \quad (6)$$

The cutoff in the integral in (6) at the upper limit is put at $R \cong L_s$, the diffusion length of conduction-electron spins, since $L_s \ll L_{T_K}$. The last condition is valid if the spin diffusion length is small compared to the localization radius ξ of conduction electrons, since

$$(L_s/L_{TK})^2 = \left(\ln \frac{L_s}{a} / \ln \frac{\xi}{a} \right) \ll 1.$$

In a wire whose transverse dimensions are less than L_0 , electron diffusion has a one-dimensional character and, for $R < L_0$,

$$K(R) = \frac{2\lambda}{\pi S_1^2 D} \ln \frac{L_0}{R}, \quad (7)$$

where S_1 is the area of the transverse cross section of the wire.

Using expressions (10) and (11) for $K(R)$, we find that

$$T_K \cong \frac{S(S+1)}{3} J^2 N_s S_1 \int_{-\infty}^{\infty} dR K(R) = \frac{2S(S+1)}{3\pi} [1 - (1-\lambda)^{3/2}] \frac{J^2 L_s N_s}{D S_1}. \quad (8)$$

We note that in a wire the condition $L_s \ll L_{TK}$ is equivalent to the conditions $L_s \ll \xi$ and $T_K \sim D/L_s \xi$.

To obtain the results enumerated above, we can use the methods developed in Refs. 3 and 4. The diffusion contribution to the electron spin correlation function at large distances due to interelectronic interaction is determined by the diagrams shown in Fig. 1. Only the interaction between electrons from different spin subbands need be taken into account here. The Pauli matrices σ_z occur at the field vertices. The interaction between electrons is represented by the wavy line. It is convenient to represent the diagram in the coordinate representation in the form shown in Fig. 2. The dot-dashed line in Fig. 2 corresponds to the diffusion, while the shaded circle corresponds to the

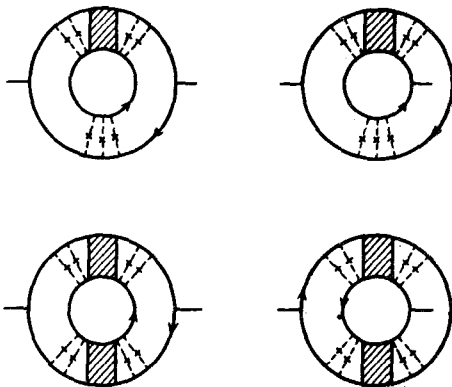
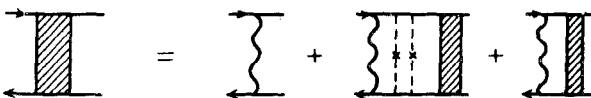


FIG. 1.



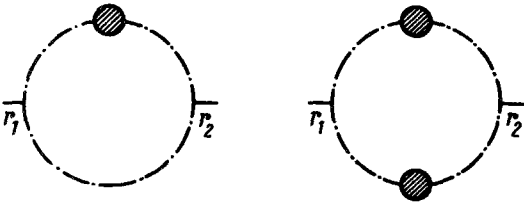


FIG. 2.

interaction between diffusions. It is evident from Fig. 2 that since the points r_1 and r_2 are connected by diffusions, whose characteristic scale is $L_0 \gg l$, the exchange between localized magnetic moments due to diffusions can have a long-range nature.

At distances larger than the mean free path, only diagrams shown in Fig. 1 contribute to the correlation function of the electron spin density, and we obtain the following expression for $K(R)$ at $R > l$:

$$\begin{aligned}
 K(R) &= 4 \int (dk) l^{iKR} T \\
 &\times \sum_{\omega_m} \int (dq) \frac{U_{q, \omega_m} |\omega_m|}{(Dq^2 + |\omega_m| + DL_s^{-2})^2 [D(q+k)^2 + |\omega_m| + DL_s^{-2}]} \times \\
 &\times \left[2 - \frac{U_{q+k, \omega_m} |\omega_m|}{D(q+k)^2 + |\omega_m| + DL_s^{-2}} \right], \\
 U_{q, \omega_m} &= \lambda \left[1 - \lambda \frac{Dq^2 + DL_s^{-2}}{Dq^2 + |\omega_m| + DL_s^{-2}} \right].
 \end{aligned} \tag{9}$$

Here $(dq) = d^d \mathbf{q} / (2\pi)^d a^{3-d}$ (d is the effective dimensionality of the specimen) and $\omega_m = 2\pi mT$. Expression (9) can be represented in the form

$$K(R) = 8 T \sum_{\omega_m > 0} \left\{ \Gamma_{\omega_m + DL_s^{-2}}^2(R) - \frac{\Gamma_{\omega_m + DL_s^{-2}}^2(R)}{1 - \lambda} \right\}, \tag{10}$$

where

$$\Gamma_{\omega_m} = \int (dq) \frac{e^{iqR}}{Dq^2 + \omega_m}. \tag{11}$$

Using (10) and (11), we can obtain expressions (3), (5), and (7) for $K(\mathbf{R})$ in specimens with different dimensionality.

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