Spin-orbit coupling of quarks and magnetic moments of baryons

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The contribution of the spin-orbit coupling of quarks to the magnetic moments of baryons is calculated. The magnetic moments predicted for the baryons by the nonrelativistic quark model with spin-orbit coupling of quarks agree well with experiment.

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1. The magnetic moments of baryons have been measured highly accurately in the past few years, and discrepancy has been found between the experimental results and the predictions of the nonrelativistic quark model. The measured magnetic moments of the Σ^- and Ξ^- hyperons differ from the predicted values by a factor of 1.5, while the discrepancy in terms of the magnetic moment of the Ξ^0 hyperon reaches 13 systematic deviations.[1,2] In this letter we show that these contradictions stem primarily from the neglect of the spin-orbit interaction of the quarks in the calculations of the magnetic moments of the baryons. The effect is analogous ot the well-known Göppert Mayer-Jensen effect in nuclear physics.[3] Just as the spin-orbit coupling of nucleons contributes to the magnetic moment of the nucleus, the spin-orbit coupling of an electron with a proton contributes to the magnetic moment of the hydrogen atom.[4] It can be seen from Table I that the agreement between theory and experiment can be improved substantially by taking into account the spin-orbit forces between quarks. We are not introducing any new parameters in the model.

2. We begin with a description of the spin-orbit interaction of quarks by means of the Breit potential, which arises in the order $(v/c)^2$ from the vector potential $V = -(4/3)(\alpha_s/r)$ (single-gluon exchange) and the scalar confinement parameter[5] $S = \nu r$:

$$U_{es} = \frac{V' - S'}{2r}\left\{\frac{\mathbf{l}_1\mathbf{s}_1}{m_1^2} - \frac{\mathbf{l}_2\mathbf{s}_2}{m_2^2}\right\} + \frac{V'}{m_1 m_2 r}\{\mathbf{l}_1\mathbf{s}_2 - \mathbf{l}_2\mathbf{s}_1\}, \tag{1}$$

where $V' = dV/dr$ and $S' = dS/dr$. After singling out the spin-orbit contribution (1) to the magnetic moment of the system, we naturally would use a nonrelativistic description in the potential model which remains. Assuming symmetry of the baryon wave function with respect to the quark coordinates, and taking into account the two-particle nature of the spin-orbit forces, we can find the total magnetic moment of the quark using potential (1):

$$\mu_1 = e_1\lambda_1 + \left\langle \frac{rV_1'}{12} - \frac{rS'}{12} \right\rangle \frac{3}{2}\,\frac{m_2 + m_3}{m_1 + m_2 + m_3}\cdot\frac{e_1}{m_1^2}$$

$$+ \left\langle \frac{rV'}{12} \right\rangle \left\{ \frac{2m_1 + m_3}{m_1 + m_2 + m_3}\,\frac{e_2}{m_1 m_2} + \frac{2m_1 + m_2}{m_1 + m_2 + m_3}\,\frac{e_3}{m_1 m_3} \right\}, \tag{2}$$

where $e_i\lambda_i$, m_i, and e_i are the intrinsic magnetic moments, masses, and charges of the quarks. The magnetic moments of the quarks and all the baryons are thus expressed in terms of the intrinsic magnetic moments of the u, d, and s quarks and in terms of the expectation values $\langle rS'\rangle$ and $\langle rV'\rangle$.

3. Appearing as parameters in potential (1) are the quantities α_s/m^2 and ν/m^2, where m is the mass of the u and d quarks. We will determine these parameters from the splitting of the energy spectrum of p-wave mesons caused by the spin-orbit interaction. To avoid difficulties with the mixing of $I = 0$ p-wave mesons, and noting that in the $I = 1$ and $I = 0$ mesons it is not possible to separate the vector spin-orbit coupling from the scalar coupling, we use data on the strange $I = 1/2$ p-wave mesons. Arguments analogous to those of Ref. 6 incorporating the mixing parameters of the Q_A and Q_B mesons and the mass splitting of the K^{**}, Q_1 and Q_2 mesons[2] lead to the values $\langle S'/m^2r\rangle = 639$ MeV and $\langle V'/m^2r\rangle = 294$ MeV. Here we have assumed $m_s = 1.5m$, in reasonable agreement with data from the spectroscopy of light mesons and baryons. The expectation values $\langle r^{-1}\rangle$ and $\langle r^{-3}\rangle$ required for determining the parameters α_s/m^2 and ν/m^2 can be calculated in the oscillator model, for example. The radial wave function of the s state in an oscillator potential is

$$\psi_s \sim \exp\left\{-\frac{1}{2}\left(\frac{r}{a}\right)^2\right\}$$

and that of the p state is $\psi_p \sim r\psi_s$. The parameter a can be determined through a normalization to the rms radius of the K meson. Corresponding calculations can be

TABLE I.

Baryon	$ls = 0$ (*n.m.*)	$ls \neq 0$ (*n.m.*)	Experimental (*n.m.*)
p	2,793	2,793	2,793
N	− 1,913	− 1,913	− 1,913
Λ	− 0,614	− 0,614	− 0,6135 (40)
ΣΛ	− 1,633	− 1,445 (4)	− 1,82 (20)
Σ^+	2,673	2,663 (4)	2,33 (13)
Σ^0	0,791	0,592 (6)	0,46 (28)
Σ^-	− 1,091	− 1,478 (15)	− 1,41 (25)
Ξ^0	− 1,436	− 1,262 (3)	− 1,250 (14)
Ξ^-	− 0,494	− 0,700 (8)	− 0,75 (6)
Ω^-	− 1,842	− 2,370 (21)	−

carried out for a linear potential for the interaction between quarks. As a result, taking into account the model dependence of the results, we find $\alpha_s/m^2 = 0.138$–0.149 GeV fm^3 and $v/m^2 = 0.678$–0.692 GeV fm. For $m = 250$ MeV we then find $v = (460\ \text{MeV})^2$ and $\alpha_s = 1$.

In calculating the expectation values $\langle rS' \rangle$ and $\langle rV' \rangle$ in (2) we assumed that they were independent of the composition of the baryons, and we used oscillator wave functions for the quarks, normalized to the rms radius of the proton. These calculations yield $\langle rS'/6m^2 \rangle = 3.44$–$3.49$ n.m. and $\langle rV'/6m^2 \rangle = 0.72$–$0.77$ n.m. We held the intrinsic magnetic moments of the *u*, *d*, and *s* quarks constant, normalizing on the basis of the most accurate measurements of the magnetic moments of the proton, the neutron, and the Λ hyperon. The results of these calculations of the baryon magnetic moments are shown in Table I; the indicated error reflects the model dependence of the predictions. Shown at the left for comparison are the predictions of the quark model without spin-orbit coupling.

We will not consider here the corrections to the magnetic moment of the system which stem from the transformation properties of the wave function of a bound state under boost transformations.[7] These corrections amount to 10–15% of the *ls* corrections and slightly improve the agreement with experiment.

4. With the exception of that for Σ^+, all the baryon magnetic moments calculated in the nonrelativistic quark model with spin-orbit coupling agree with experiment within the experimental error. Interestingly, the magnetic moment of Ξ^0 also lies

within the experimental error. The correction to the magnetic moment of Σ^+ has the correct sign, but there is still a discrepancy amounting to two or three systematic deviations. We have shown in Table I the transition magnetic moment for the decay $\Sigma \to \Lambda\gamma$. The energy released in this decay is about 80 MeV, and with $m = 250$ MeV this energy represents about a third of the mass of a constituent quark. We should therefore expect that the static approximation would be correct within $\sim 30\%$. The theory agrees with experiment within this accuracy. We have also shown here predictions of the magnetic moment of the Ω^- hyperon, a quasistable particle.

On the whole, the nonrelativistic potential model with spin-orbit coupling of the quarks agrees well with experiment. Further refinement of the model will require more accurate experimental data on the magnetic moments of the baryons.

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