Chaotic inflating universe

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It is shown that inflation is a natural result of chaotic initial conditions in the early universe. These initial conditions are found in a wide class of elementary particle theories.

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It now seems probable that the new scenario of an inflating universe[1] (see also Ref. 2) can be completely realized within the framework of $N = 1$ supergravitation.[3] This circumstance could be of great significance, since this scenario permits solving simultaneously many different cosmological problems, such as the problem of the horizon, flatness, uniformity, and isotropy of the universe, the problem of primordial monopoles and the gravitino, etc.[4] At the same time, in developing this scenario,[1] the impression has been created that the inflation (prolonged exponential expansion) of the universe is a quite exotic phenomenon, which arises due to strong supercooling during phase transitions in the early universe and which is realized only in a very limited class of theories.[1-4] In this paper, we shall demonstrate that this is not so and that under certain natural assumptions about the initial conditions in an expanding universe, inflation arises in a natural manner in a wide class of relativistic theories.

We shall examine as an example the theory with effective potential $V(\phi) = (\lambda/4)\phi^4$ without the term $\sim R\phi^2$, where R is the scalar curvature). It is usually assumed that the universe during the period $t \lesssim t_p \sim M_p^{-1} \sim (10^{19}\ \mathrm{GeV})^{-1}$ from the beginning of expansion was in a chaotic state with the curvature tensor varying rapidly from point to point, and it is only later that the universe became uniform and isotropic.[5] Let us assume that the magnitude of the classical field ϕ at first also had different random values at different points in space,and let us follow the evolution of the field ϕ in time. We are interested in the regions of space where the field ϕ, for accidental reasons, was quite uniform. If the size of the corresponding region at first exceeded the size of the horizon in the de Sitter universe $\sim 2H^{-1}$, where H is Hubble's constant during that epoch

$$H = \sqrt{\frac{8\pi V(\phi)}{3M_p^2}} = \sqrt{\frac{2\pi\lambda}{3}}\,\frac{\phi^2}{M_p}, \tag{1}$$

then the interior of this region expanded according to the law

$$a(t) = a_0 e^{Ht}, \tag{2}$$

which is nearly independent of the expansion of the rest of the universe. The equation of motion of the field ϕ inside such a region is

$$\phi + 3H\phi = -\lambda\phi^3. \tag{3}$$

Hence, it follows that for $\phi^2 \gg M_p^2/6\pi$

$$\phi = \phi_0 \exp\left(-\frac{\sqrt{\lambda}\,M_p}{\sqrt{6\pi}}t\right). \tag{4}$$

This means that the characteristic time over which the field ϕ decreased appreciably is of the order of $\Delta t \sim \sqrt{6\pi}/\sqrt{\lambda}\,M_p$. Over this time, the scale factor $a(t)$ in this region increases considerably,

$$a(\Delta t) \sim a_0 e^{H\Delta t} \sim a_0 e^{2\pi\phi_0^2/M_p^2}. \tag{5}$$

In order that the region with dimensions $l \gtrsim H^{-1}$ expand up to dimensions exceeding the dimensions of the observed part of the universe, it is sufficient that the quantity $a(\Delta t)$ exceed $\sim a_0 e^{65}$ (Ref. 1–4 and 6); hence,

$$\phi_0 \gtrsim 3M_p. \tag{6}$$

The existence of fields with such amplitude in the early universe is entirely acceptable. The only possible restriction on the amplitude of the field is the requirement that the quantum-gravitational effects $V(\phi) \lesssim M_p^4$, be small. This condition is satisfied at

$\lambda \lesssim 10^{-2}$ in the $(\lambda/4)\phi^4$ theory for $\phi \gtrsim 3M_p$. This is, however, a relatively weak restriction. In addition, it is usually assumed that in the early universe near the singularity the total energy density (including $V(\phi)$) approached infinity,[5] so that the condition $V(\phi) \lesssim M_p^4$ does not appear to be necessary. Of course, very strong restrictions remain on $V(\phi)$, which are attributable, for example, to the fact that it is necessary to obtain density inhomogenities with amplitude $\delta\rho/\rho \sim 10^{-4}$ after inflation.[7,8] Nevertheless, we believe that it is important that the inflation itself can be realized in a much wider class of theories than that expected in Refs. 1–4.

An investigation of the role of high-temperature effects in our scenario and estimates of the probability of the appearance of regions with dimensions $l \gtrsim 2H^{-1}$, containing a uniform field $\phi \gtrsim 3M_p$ will be reported in a separate paper.[9] Here it is important only that the probability for the appearance of such regions per unit volume does not equal zero. For this reason, in the open (infinite) universe there must be an infinite number of such regions, each of which after inflation (and subsequent expansion) becomes a separate miniuniverse with dimensions exceeding the dimension of the observed part of the universe $l \sim 10^{28}$ cm. In this respect, the proposed scenario differs strongly from other approaches to the theory of a chaotic universe, in which it is usually required that the entire universe, after expansion, be homogeneous and isotropic. As is now clear, this requirement, which at first glance seemed logical, is too strict.

It should also be noted that the scenario of chaotic inflation differs strongly from all remaining variants of the inflating universe scenario, since the proposed scenario is not based on a study of high-temperature phase transitions in theories with spontaneous symmetry breaking.

In conclusion, we note that the results obtained above and the estimates concerning the $V(\phi) = (\lambda/4)\phi^4$ theory are largely model independent. Indeed, according to Refs. 1 and 4, the basic condition for strong inflation is $H^2 \gtrsim 20m^2(\phi)$, where $H^2 = (8\pi/3)[V(\phi)/M_p^2]$ and $m^2(\phi) = d^2V/d\phi^2$. For large ϕ, the quantity $m^2(\phi)$ can be estimated simply as V/ϕ^2. Hence, it follows that strong inflation is realized in regions with $\phi \gtrsim 2M_p$(6) in a wide class of theories in which such an estimate is valid. Thus the expansion of the universe is not an exotic phenomenon, which is possible only in some special models of the Coleman-Weinberg type,[1–2] but is a natural consequence of chaotic initial conditions in the early universe, which are realized in a wide class of elementary particle theories.

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