Anderson transition in a disordered quasi-one-dimensional system

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All the states remain localized in the transverse-coupling region $w < 1/\tau \ll \varepsilon_F$ when the diagrams with maximally intersecting impurity lines are taken into account in a self-consistent manner. Diffusion occurs at $w\tau \gtrsim 1$. The frequency dependence of the conductivity is calculated.

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The problem of the localization of a quantum particle in a disordered one-dimensional system can be studied exactly by the Berezinskiĭ method,[1] but attempts to apply this method to two- or three-dimensional systems run into serious mathematical difficulties.[2] Some diagram methods have recently been developed for a qualitative study of the electron-localization problem in systems of dimensionality $d = 1,2$ (Refs. 3–5), and 3 (Refs. 6 and 7) (Anderson localization). A qualitative agreement with the exact results has been achieved in the $d = 1$ case.[1] In the present letter we take this approach to study a quasi-one-dimensional system. The Green's function of the electron can be written

$$G_{\pm}^{-1}(\omega, p) = \omega - v_F(|p_{\|}| - p_F) + w\phi(p_{\perp}) \pm i/2\tau ;$$
$$\phi(p_{\perp}) = \cos ap_x + \cos ap_y ; \tag{1}$$
$$\frac{1}{\tau} = 2\pi u^2 N(0) ; \quad N(0) = \frac{1}{\pi v_F a^2} .$$

We are assuming $\varepsilon_F \gg w,\ 1/\tau$.

Gor'kov *et al.*[3] have reported some diagrams which cause a substantial renormalization of the diffusion coefficient for $d = 1,2$; these diagrams are shown in Fig. 1

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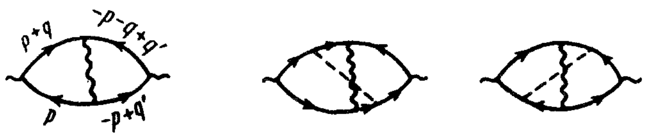

FIG. 1.

(Refs. 3–5). Here a wavy line is a diffusion in the electron-electron channel, where its Green's function is of the standard form:

$$D^0(q,\omega) = \frac{u^2\tau^{-1}}{-i\omega + D^0_{\|}\, q^2_{\|} + D^0_{\perp}\,(2-\phi(q_{\perp}))} \tag{2}$$

$$D^0_{\|} = v_F^2\tau\,; \qquad D^0_{\perp} = w^2\tau.$$

The corresponding corrections to the diffusion which result from these diagrams are given in the quasi-one-dimensional case by

$$D_{\|}(\omega) = D^0_{\|} - \frac{1}{\pi N(0)}\int \frac{d^3q}{(2\pi)^3}\,\frac{\widetilde{D}_{\|}}{-i\omega + \widetilde{D}_{\|}q^2_{\|} + \widetilde{D}_{\perp}(2-\phi(q_{\perp}))}\,, \tag{3}$$

$$D_{\perp}(\omega) = D^0_{\perp} - \frac{1}{\pi N(0)}\int \frac{d^3q}{(2\pi)^3}\,\frac{D_{\perp}}{-i\omega + \widetilde{D}_{\|}q^2_{\|} + \widetilde{D}_{\perp}(2-\phi(q_{\perp}))}\,, \tag{4}$$

where $\widetilde{D} = D_0$. The integration over $q_{\|}$ in (3) and (4) is restricted by the condition[3–7] $q_{\|} \leqslant (D^0_{\|}\tau)^{-1/2} = 1/l_0$, since diffusion always occurs over distances greater than the mean free path l_0. The integration over $q_{\perp}$ in (3) and (4) is carried out over the entire first Brillouin zone if $(\omega\tau)^2 < 1$. It follows from Eqs. (3) and (4) that, as in the cases $d = 1,2$, the corrections may prove extremely important at low values of the parameter w (more on this below), and we are confronted with the unresolved problem of taking into account the succeeding corrections—a thoroughly complicated problem.[3–6] Following Ref. 4, we take the sum of the corrections of this sort in a self-consistent way: We set $\widetilde{D} = D(\omega)$ in Eqs. (3) and (4). As a result, we find

$$\frac{D_{\|}(\omega)}{D^0_{\|}} = \frac{D_{\perp}(\omega)}{D^0_{\perp}} = \alpha(\omega),$$

$$\alpha = 1 - \chi_1 + \frac{-i\widetilde{\omega}}{\alpha}\chi_2\,, \tag{5}$$

where

$$\chi_1 = \int_0^1 \frac{dq}{\pi}\int \frac{d^2q_{\perp}}{(2\pi)^2}\,\frac{1}{q^2 + \widetilde{w}^2(2-\phi(q_{\perp}))}\,, \tag{6}$$

$$\chi_2 = \int_0^1 \frac{dq}{\pi}\int \frac{d^2q_{\perp}}{(2\pi)^2}\,\frac{1}{q^2 + \widetilde{w}^2(2-\phi)}\,\frac{1}{-i\,\widetilde{\omega}/\alpha + q^2 + \widetilde{w}^2(2-\phi)}\,, \tag{7}$$

We have introduced $\tilde{w} = w\tau$ and $\tilde{\omega} = \omega\tau$. The quantity χ_1 in (6) is a monotonic function of $\tilde{w}$: $\chi_1(\tilde{w}\to 0) \propto 0.37/\tilde{w}$ and $\chi_1(\tilde{w}\to\infty) \propto \ln \tilde{w}/(\pi\tilde{w})^2$. There accordingly exists a critical value[1)] $\tilde{w}$, given by $\tilde{w}_c \cong 0.31$, where $D(w=0)=0$. Near the threshold, Eq. (5) can be rewritten $[|\tilde{\omega}/\alpha| \ll 1, |\varepsilon| \ll 1; \varepsilon = (w - w_c)/w_c]$

$$\alpha = 1,25\,\epsilon + 1,64\sqrt{-i\tilde{\omega}/\alpha}\,. \tag{8}$$

It follows that at $w > w_c$ the system is in the diffusion regime. Setting $\sigma(\omega) = \alpha(\omega)\sigma_0$ we find, for $w > w_c$

$$\frac{\sigma_{dc}}{\sigma_0} = \epsilon\,; \quad \frac{\sigma_{ac}}{\sigma_0} \sim \begin{cases} (1-i)|\tilde{\omega}/\epsilon|^{1/2}\,; & |\tilde{\omega}| \ll |\epsilon|^3 \\ (\sqrt{3}-i)|\tilde{\omega}|^{1/3}\,; & |\epsilon|^3 \ll \tilde{\omega} \ll 1 \end{cases}\,. \tag{9}$$

At $w < w_c$, as in the one-dimensional case, all the states are localized, and we have

$$\frac{\sigma(\omega)}{\sigma_0} \sim \begin{cases} \dfrac{-i\tilde{\omega}}{|\epsilon|^2} + \dfrac{2|\tilde{\omega}|^2}{|\epsilon|^5}\,; & |\tilde{\omega}| \ll |\epsilon|^3 \\ (\sqrt{3}-i)|\tilde{\omega}|^{1/3}\,; & |\epsilon|^3 \ll \tilde{\omega} \ll 1 \end{cases}\,. \tag{10}$$

The static conductivity in this range of w may have a hopping mechanism. Since we are dealing with the case of weak localization, this conductivity is

$$\sigma_{hop} \sim \frac{e^2 n}{kT}\,\nu(T)\,\lambda^2\,, \tag{11}$$

where λ is the localization radius,

$$\frac{\lambda_{\parallel}}{\sqrt{D_{\parallel}^0 \tau}} \sim \frac{\lambda_{\perp}}{\sqrt{D_{\perp}^0\,\tau a^2}} \sim 1/\epsilon \tag{12}$$

and $\nu(T)$ is the hopping frequency, which depends on the particular mechanism that causes the hops. We are assuming[8] $\varepsilon \gg \tau\nu$.

Scaling relations (9) and (10) correspond to a three-dimensional Anderson transition.[6,7] The specific one-dimensional dependence[1,4] $\sigma(\omega)$ holds at

$$\sigma/\sigma_0 \sim -i\tilde{\omega} + 8\tilde{\omega}^2$$

and at $w > w_c$ only at high frequencies, $\omega\tau > 1$, since $w_c \sim 1/\tau$. In contrast, in the quasi-two-dimensional case the low-frequency two-dimensional dependence $\sigma(\omega)$ may be manifested even at $E_F > E_c$.

It should be noted that the approximation of Ref. 9 corresponds to neglecting the normalization of $D_\perp$ in Eqs. (3) and (4), and in this case there is no threshold along the w scale. As shown above, however, even the simplest rules for summing the corrections in w to $D_\perp^0$ by the scheme proposed in Ref. 4 give rise to a localization threshold.

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[1)] In the quasi-two-dimensional case the situation is different. Here the introduction of a finite w immediately gives rise to a region of delocalized states, $E > E_c = 1/\pi\tau \ln(1/\sqrt{2}\, w\tau)$.

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