Geometric resonance in low-frequency conductivity of one-dimensional metals

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An oscillatory dependence of the dissipative 1*d* conductivity of a metal on the wave number or frequency of the external wave is predicted. The physical mechanism of this phenomenon is related to the existence of a fixed macroscopic length L, equal to the hopping distance of an electron between two localized states, such that the transition matrix element oscillates as a function of the increase in phase of the wave over the length L.

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It is well known that at $T = 0$ the static conductivity of 1*d* conductors is zero due to the localization of all electronic states. According to Refs. 1 and 2, a finite conductivity $\mathrm{Re}\,\sigma(\omega) \sim \omega^2 \ln^2 \omega$ appears in an alternating uniform field. The effect of spatial dispersion on the conductivity is yet to be studied. Although Berezinskiĭ's theory in principle permits taking into account the dependence of σ on the wave number q, the problem of solving the equations[2] with finite q turns out to be very difficult.

The complex conductivity of a 1*d* metal for all ω and q has the form

$$\sigma(\omega, \kappa) = - i\,\omega\,\sigma_0 \int_0^{\infty} dt\, e^{-2i\omega t}\,[\,y_{\kappa,\omega}(t) + y_{-\kappa,\omega}(t)\,], \tag{1}$$

where $\sigma_0 = ne^2\tau/m$, n is the electron density, e is the electron charge, and m is the effective mass of the electron, τ is the back-scattering time, ω and κ are the dimensionless frequency and wave number in units of $1/\tau$ and $1/l$ in addition, $l = v\tau$, and v is the Fermi velocity. The function $y_{\kappa,\omega}(t)$ satisfies the equation[2]

$$\left\{\frac{d}{dt}\left[t(1+t)\frac{d}{dt}\right]+2i\omega t\frac{d}{dt}\;(1+t)+i(\omega-\kappa)\right\}y_{\kappa,\omega}(t)=2i\omega\int_0^\infty dx\,\frac{e^{-2i\omega x}}{1+t+x} \tag{2}$$

with the boundary conditions that it remains finite at the origin and decrease in the limit $t\to+\infty$.

We shall present the results of the solution of Eq. (2) and calculations of $\sigma(\omega,\kappa)$ for finite κ in different limiting cases, for low frequencies $\omega\ll 1$.

At $\kappa\ll 1$

$$\frac{\mathrm{Re}\,\sigma(\omega,\kappa)}{\sigma_0}=2\left(\frac{\omega}{\kappa}\right)^2\mathcal{F}(\kappa);\qquad \mathcal{F}(\kappa)=1-e^{-\kappa^2 z}(\cos\kappa z+\kappa a\sin\kappa z)\,, \tag{3}$$

$$z=2\,|\ln\frac{\omega}{2}\,|\,;\qquad a=2(C+\ln 4-1);\qquad C=0{,}577\ldots\;. \tag{4}$$

This expression under the condition that $\kappa z\ll 1$ goes over into Berezinskiĭ's result.[2] In the region $\kappa^2 z<1<\kappa z$, the conductivity oscillates as a function of κz:

$$\frac{\mathrm{Re}\,\sigma(\omega,\kappa)}{\sigma_0}=(2\omega)^2\left(\frac{\sin^2\frac{\kappa z}{2}}{\kappa^2}+\frac{z}{2}\right). \tag{5}$$

The period of oscillations with respect to κ is $2\pi/z$, the relative amplitude is of the order of $2/\kappa^2 z$, and the value at the minimum is $2\omega^2 z$. In the region $z^{-1}\lesssim\kappa^2<1$, the oscillating terms decrease exponentially, and $\mathrm{Re}\sigma(\omega,\kappa)=2\sigma_0(\omega/\kappa)^2$. Finally, at $\kappa\gg 1$ we have $\mathrm{Re}\sigma(\omega,\kappa)=2\sigma_0\omega^2/\kappa^4$.

The quasiharmonic oscillations of the conductivity, described by Eq. (5), represent the geometric resonance effect. These oscillations reveal the characteristic electron length $L=2t\,|\ln(\omega/2)|$, which determines the spatial period of oscillations of absorption in the nonuniform field of the wave. We shall show that this length can be interpreted as a fixed (by the frequency ω) hopping length of an electron between two localized states, whose energies differ by ω. In accordance with Ref. 1, in order of magnitude,

$$\frac{\mathrm{Re}\,\sigma(\omega,\kappa)}{\sigma_0}\simeq|D(\omega,\kappa)|^2, \tag{6}$$

where $D(\omega,\kappa)$ is the dimensionless Fourier transform of the matrix element of the momentum operator

$$D(\omega,\kappa)=\frac{1}{2i}\int_{-\infty}^{\infty}d\xi\,\psi_{\epsilon+\omega}(\xi)\left(e^{i\kappa\xi}\frac{d}{d\xi}+\frac{d}{d\xi}e^{i\kappa\xi}\right)\psi_\epsilon(\xi) \tag{7}$$

between the wave functions of the localized states (ε is the Fermi energy):

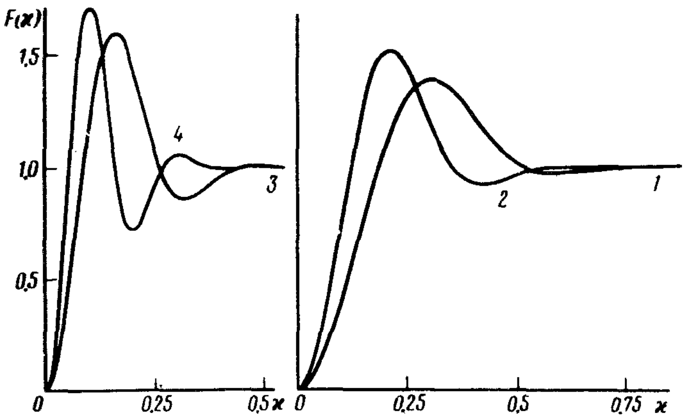

FIG. 1. Dependence of the dissipative conductivity of a 1d metal on $\kappa = ql$ for difference $\omega\tau$: $1 - \omega\tau = 10^{-2}$, $2 - \omega\tau = 10^{-3}$,$3 - \omega\tau = 10^{-4}$, $4 - \omega\tau = 10^{-6}$.

$$\psi_\epsilon(\xi) = \exp\left(-\frac{|\xi|}{2}\right); \qquad \psi_{\epsilon+\omega}(\xi) = \exp\left(-\frac{|\xi - z|}{2}\right). \tag{8}$$

The maxima in these functions are separated in z, since their energies differ by an amount $\omega \ll 1$.[1] For small κ, the region $0 < \xi < z$ makes the main contribution to (7), and we obtain

$$|D(\omega, \kappa)|^2 = e^{-z} \frac{\sin^2 \frac{\kappa z}{2}}{\kappa^2} = \left(\frac{\omega}{2\kappa}\right)^2 \sin^2 \frac{\kappa z}{2}. \tag{9}$$

This result coincides with the first term in Ref. 5 to within a numerical factor.

Thus the geometrical resonance in the low-frequency conductivity of 1d metals is due to the oscillatory dependence of the transition matrix element on the increase in the phase of the wave over the hopping distance of the electron.

Figure 1 shows the computed dependence of the function $\mathscr{F}(\kappa)$ for different ω. The first oscillation of the conductivity is clearly evident and, for sufficiently low ω, a second oscillation appears. The subsequent periods are not visible, because they fall into the region of values of κ where the amplitude of the oscillating term is very small.

This effect can probably be observed in the absorption of slow (compared to v) waves at low temperatures $kT \ll \hbar\omega$ in 1d metals, in which the Peierls transition is missing.

[1]N. Mott and E. A. Davis, Electronic Processes in Noncrystalline Materials, Oxford University Press, New York, 1970.
[2]V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **65**, 1251 (1973) [Sov. Phys. JETP **38**, 620 (1974)].

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