Fast annihilation of oppositely directed magnetic fields in a plasma

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The calculated rate at which oppositely directed magnetic fields annihilate is dramatically increased by incorporating the heat capacity and emission of the plasma.

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The current sheets near a magnetic field null in a plasma play an imporant role in astrophysics and in plasmas in the laboratory. There may be a rapid dissipation of the energy of the magnetic field here, as is seen, for example, in solar flares.[1] The diffusion of the magnetic field across a current sheet determines the lifetime of plasmas with reversed magnetic fields, which are of interest in the controlled fusion program.[2]

The conventional way to calculate the rate of the magnetic field diffusion is to assume that in a plasma of density n_0 there is a magnetic field H_z equal to H_0 at $x > 0$ and equal to $-H_0$ at $x < 0$. The plasma pressure is assumed negligibly small in comparison with the magnetic pressure. In a transition layer the magnetic energy is expended on the Joule heating of electrons, whose temperature accordingly rises to $T_0 \sim H_0^2/8\pi n_0$. The thickness of the current sheet, Δx, increases in accordance with $\Delta x \sim (D_0 t)^{1/2}$, where $D_0 = c^2/4\pi\sigma_0$ is the magnetic viscosity coefficient, which is determined by the plasma conductivity $\sigma_0 = \sigma(T_0)$. Because of the high conductivity of the hot plasma, the rate of annihilation of magnetic fluxes of different signs calculated by this approach turns out be far smaller than the observed rate. Accordingly, attempts to explain the mechanism for solar flares use models with a more complex magnetic field configuration, where a so-called reconnection of magnetic lines of force can occur.[1] In systems with a reversed field, on the other hand, it is assumed that the rapid diffusion of the magnetic field results from the anomalous resistance of the plasma.[2]

We wish to point out that the rate of annihilation of oppositely directed fields may increase markedly when heat transfer away from the neutral sheet is taken into account. This heat transfer may result, in particular, from the thermal conductivity, since in an isothermal $(T_i \approx T_e = T)$ magnetized plasma the transverse thermal diffusivity[3] $\chi_\perp \sim r_{Hi}^2 \nu_i$ is $(m_i/m_e)^{1/2}$ times larger than the magnetic viscosity at $nT \sim H^2/8\pi$.

We start from the plasma transport equation,[3] omitting all the terms irrelevant to this problem:

$$nT + H^2/8\pi = H_0^2/8\pi ,$$

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} - vH\right),$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT + \frac{H^2}{8\pi}\right) = \frac{\partial}{\partial x}\left\{\kappa_\perp \frac{\partial T}{\partial x} + \frac{H}{4\pi}\left(\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} - vH\right)\right\},$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0 .$$

These equations have a self-similar solution, so that all quantities depend only on the variable $\xi = x/(D_0 t)^{1/2}$. After introducing the dimensionless quantities

$$H(x,t) = H_0 H(\xi), \quad n(x,t) = n_0 n(\xi),$$

$$T(x,t) = T_0 T(\xi), \quad v(x,t) = (D_0/t)^{1/2} v(\xi)$$

we can rewrite the basic equations as

$$nT + H^2 = 1, \tag{1}$$

$$-\frac{\xi}{2}\frac{dH}{d\xi} = \frac{\partial}{\partial \xi}\left(T^{-3/2}\frac{dH}{d\xi} - vH\right), \tag{2}$$

$$-\frac{\xi}{4}\frac{d(nT)}{d\xi} = \frac{d}{d\xi}\left\{\mu^{-1/2}\frac{n^2}{H^2 T^{1/2}}\frac{dT}{d\xi} + \frac{H}{4}\left(T^{-3/2}\frac{dH}{d\xi} - vH\right)\right\}, \tag{3}$$

$$\frac{\xi}{2}\frac{dn}{d\xi} = \frac{d}{d\xi}(nv), \tag{4}$$

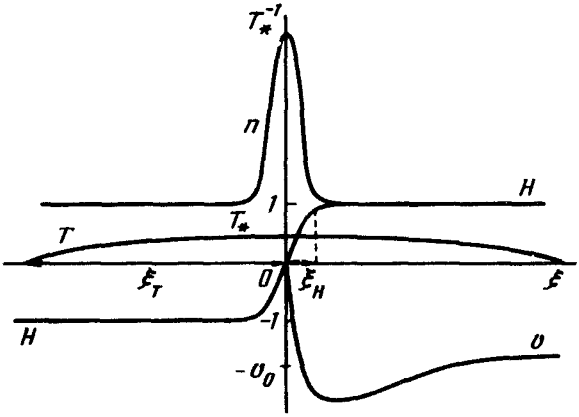

FIG. 1. The magnetic field (H), the density (n), the temperature (T), and the flow velocity of the plasma (v) at a neutral sheet.

where $\mu \equiv m_e/m_i \ll 1$.

A high thermal conductivity of the plasma causes a qualitative change in the solution (Fig. 1). The solution becomes a two-scale solution, with the scale dimension for changes in the magnetic field, ξ_H, much smaller than the temperature scale dimension ξ_T. The plasma temperature at the neutral sheet, T_*, decreases significantly: $T_* \ll 1$. It then follows immediately from the pressure balance condition, (1), that the plasma at the neutral sheet is greatly compressed, to a density $n_* \sim T_*^{-1} \gg 1$. This compression results from the flow of plasma from the periphery toward the neutral sheet at a velocity v_0. The parameters T_*, ξ_T, ξ_H, and v_0 can be determined in order of magnitude from the following considerations. The plasma temperature T_* and the scale dimension for temperature changes, ξ_T, reach values such that the conduction heat flux is comparable in magnitude to the Poynting energy flux:

$$\mu^{-1/2}\, T_*^{-1/2} \frac{dT}{d\xi} \sim \mu^{-1/2}\, T_*^{1/2}\, \xi_T^{-1} \sim v_0 H^2 \sim v_0 \,. \tag{5}$$

Integrating (4), we find yet another relation:

$$-\frac{1}{4} \int_0^\infty \xi \frac{d(nT)}{d\xi}\, d\xi = v_0/4\,, \quad \text{or} \quad T_* \xi_T \sim v_0 \,. \tag{6}$$

Since the material field is frozen in the plasma outside the neutral sheet, of thickness ξ_H, a magnetic flux equal to v_0 is carried along with the plasma flow toward the neutral sheet. At the sheet itself, where the plasma velocity decreases, the oppositely directed magnetic fluxes annihilate as a result of magnetic diffusion, so that there we can write

$$T_*^{-3/2} \frac{dH}{d\xi} \sim T_*^{-3/2}\, \xi_H^{-1} \sim v_0 \,. \tag{7}$$

The last condition follows from continuity equation (4) and means that the incoming plasma flow accumulates at the neutral sheet:

$$\frac{1}{2}\int_0^\infty \xi \frac{dn}{d\xi} d\xi \sim n_* \xi_H \sim T_*^{-1} \xi_H \sim v_0 . \tag{8}$$

From (5)–(8) we thus find

$$T_* \sim \mu^{1/8}, \quad \xi_H \sim \mu^{-1/32}, \quad \xi_T \sim \mu^{-9/32}, \quad v_0 \sim \mu^{-5/32}. \tag{9}$$

It follows that the magnetic fluxes annihilate with an effective magnetic diffusion coefficient

$$D_{ef} \sim v_0^2 D_0 \sim \mu^{-5/16} D_0 \gg D_0 . \tag{10}$$

We might note that the accelerated dissipation of magnetic energy results from not only the decrease in the plasma temperature (and the corresponding decrease in the conductivity) at the neutral sheet [as can be seen from the condition $D_{ef} \gg D(T_*) \sim T_*^{-3/2}$] but also from the pronounced compression of the neutral sheet by the plasma flow incident on it ($\xi_H \ll D_*^{1/2}$).

It follows from (10) that in a hydrogen plasma the Coulomb ion thermal conductivity raises the effective magnetic viscosity of the plasma by about an order of magnitude. This effect may be considerably stronger in the case of an anomalously high thermal conductivity (because of a plasma turbulence, for example). With a Bohm thermal conductivity for the plasma ($\kappa_\perp \sim ncT/eH$), for example, we have $D_{ef} \sim D_0(\omega_{He}\tau_e)_0^{5/11}$.

Yet another possible mechanism for heat removal from the neutral sheet may be bremsstrahlung of the plasma (the corresponding volume power level is $Q \propto n^2 T^{1/2}$). The bremsstrahlung becomes effective when the neutral sheet expands to a size $\Delta x \sim (D_0\tau_0)^{1/2}$, where $\tau_0 \sim n_0 T_0 / Q(n_0, T_0)$ is the scale radiation time. The Joule heating of the plasma is then balanced by the radiation, and the thickness of the neutral sheet remains constant, since we have $D \propto T^{-3/2}$ and $\tau_r \propto T^{3/2}$ at a constant $nT \sim H_0^2/8\pi$. The influx of plasma to the neutral sheet, on the other hand, increases the density (and correspondingly reduces the plasma temperature):

$$\Delta x \frac{dn}{dt} \sim n_0 v . \tag{11}$$

It follows from the energy balance condition $vH_0^2/4\pi \sim Q\Delta x$ that the plasma flow velocity is $v \sim (D_0/\tau_0)^{1/2}(n/n_0)^{3/2}$. Also using (11), we find

$$v \sim (D_0/\tau_0)^{1/2} (1 - t/2\tau_0)^{-3}. \tag{12}$$

The magnetic field annihilation in this case is thus explosive in nature, and its velocity can become comparable to the Alfvén velocity in a few radiation times.

A rapid decay of the plasma configuration with a reversed magnetic field caused by increased emission from the plasma due to impurities has been observed experimentally.[4]

[1]S. A. Kaplan, S. B. Pikel'ner, and V. N. Tsytovich, Fizika plazmy solnechnoĭ atmosfery (Physics of the Plasma of the Solar Atmosphere), Nauka, Moscow, 1977.

[2]Plasma Physics and Controlled Nuclear Fusion Research, IAEA, Vienna, 1981.
[3]S. I. Braginskiĭ, in Voprosy teorii plazmy, Vol. 1, Atomizdat, Moscow, 1963, p. 183 (Reviews of Plasma Physics, Vol. 1, Plenum, New York, 1965).
[4]R . Kh. Kurtmullaev *et al.*, Proceedings of the Ninth IAEA Conference on Plasma Physics, Baltimore 1982.

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